A STOCHASTIC AND ASYMMETRIC-INFORMATION FRAMEWORK
FOR A DOMINANT-MANUFACTURER SUPPLY CHAIN

Keywords: Supply Chain, Pricing, Stackelberg Game, Information Asymmetry.

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ABSTRACT

Consider a dominant manufacturer wholesaling a product to a retailer, who in turn retails it to the consumers at $p/unit. The retail-market demand volume varies with $p$ according to a given demand curve. This basic system is commonly modeled as a manufacturer-Stackelberg ([mS]) game under a “deterministic and symmetric-information” (“det-sym-i”) framework. We first explain the logical flaws of this framework, which are: (i) the dominant manufacturer-leader will have a lower profit than the retailer under an iso-elastic demand curve; (ii) in some situations the system’s “correct solution” can be hyper-sensitive to minute changes in the demand curve; (iii) applying volume discounting while keeping the original [mS] profit-maximizing objective leads to an implausible degenerate solution in which the manufacturer has dictatorial power over the channel. We then present an extension of the “stochastic and asymmetric-information” (“sto-asy-i”) framework proposed in Lau and Lau (2005), coupled with the notion that a profit-maximizing dominant manufacturer may implement not only [mS] but also “[pm]” -- i.e., using a manufacturer-imposed maximum retail price. We show that this new framework resolves all the logical flaws stated above. Along the way, we also presented a procedure for the dominant manufacturer to design a profit-maximizing volume discount scheme using stochastic and asymmetric demand information.

Using our sto-asy-i framework to resolve the logical flaws of the det-sym-i framework also reveals two noteworthy points: (i) the attractiveness of the perfectly legal but overlooked channel-coordination mechanism [pm]; and (ii) volume discounting as a means for the dominant manufacturer to benefit from information known only to the retailer.

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Please Note: Tables are given on pages 18 to 20. Appendices are on pages 20 to 26.
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1. INTRODUCTION

Consider the following basic “building block” of many two-echelon supply-chain and market channel models: a manufacturer wholesales a product at $w/unit to a retailer, who in turn retails it at $p/unit to the consumers. The retail-market demand volume $V$ varies with retail price $p$ according to a given deterministic demand curve $D_p$. How would or should the “players” (i.e., the manufacturer and the retailer) set their $w$ and $p$? An increasingly popular approach in answering this question is to assume that the players interact in the form of a non-cooperative game, and the most popular game assumed in the supply-chain literature is the “manufacturer-Stackelberg” (hereafter “[mS]”) game, where the manufacturer is assumed to be the game’s leader. See, e.g., Arcelus & Srinivasan (1987), Ertek & Griffin (2002), Li et al. (2002), Parlar & Wang (1994), Weng (1995a, 1995b), among numerous others. This paper extends two earlier papers by Lau & Lau (hereafter “LL”, 2003, 2005) in demonstrating the following:

(i) The popular two-echelon framework based on a deterministic and symmetric-information manufacturer-Stackelberg ([mS]) game has some very basic logical flaws;

(ii) These logical flaws are resolved by using our “stochastic and asymmetric-information” (hereafter “sto-asy-i”) framework, under which the dominant manufacturer can dictate a maximum permissible retail price instead of playing the [mS] game. We will show that our framework leads to much more plausible behavior and results.

This paper also reveals the strength and practicality of a legitimate channel-coordinating device that has been largely overlooked hitherto; namely, using a manufacturer-imposed maximum permissible retail price $p_M$.

This paper assumes that the reader is familiar with Lau & Lau (“LL”, 2003, 2005).

Section 2 briefly summarizes the problematic characteristics of the “deterministic and symmetric-information” (hereafter “det-sym-i”) framework as pointed out in LL, while §3 points out yet another logical dilemma of this framework. Section 4.1 summarizes the sto-asy-i framework presented in LL (2003), while §4.2 presents a numerical illustration that motivates and facilitates the extensions presented in this paper on the sto-asy-i framework. Section 5 presents results on applying LL’s (2003) sto-asy-i framework to an iso-elastic curve; these results show how our sto-asy-i framework can resolve the two problematic characteristics summarized in §2. Sections 6 and 7 present results on applying the sto-asy-i
framework to designing a volume (or “quantity”) discounting scheme; these results show that our sto-asy-i framework can resolve the logical dilemma we pointed out in §3 of this paper. These results also demonstrate clearly the strength of using a manufacturer-prescribed $p_M$ as a channel-coordinating device; hence, we discuss briefly in Appendix 1 the legal aspects of this $p_M$ arrangement. In the concluding Section 8 we summarize the main differences between our sto-asy-i framework and the classical det-sym-i framework. Review of much of the relevant literature has been done in LL (2003, 2005) and will not be repeated here.

2. SUMMARY OF THE SYSTEM AND THE BASIC KNOWN RESULTS

2.1 Basic Definitions of the System to be Considered

Define:

- $\Pi = \text{Deterministic profit.}$ $\Pi$ may have a subscript, which may be either M (for manufacturer), R (for retailer), C (for channel, i.e., “manufacturer’s plus retailer’s”), or I (for the Integrated firm doing both manufacturing and retailing). The same subscripting convention is also used (when needed) for the other variables defined below.
- $\tilde{\Theta} = \text{Stochastic profit, with the same subscripting convention as } \Pi \text{ above.}$
- $c = \text{unit manufacturing cost incurred by the manufacturer;}
- w \text{ or } w_M = \text{unit wholesale price charged by the manufacturer to the retailer;}
- p \text{ or } p_R = \text{retailer-determined unit retail price that the retailer charges the end consumers;}
- p_M = \text{a manufacturer-imposed maximum allowable unit retail price;}
- V = \text{volume (i.e., quantity) per “unit period” (say, per year) that the retailer orders from the manufacturer and sells to the consumers.}$

For any random variable $\tilde{x}$, its expected value is the bolded $x$ and its standard deviation is $\sigma_x$. Thus, $\Theta$ denotes $E(\tilde{\Theta})$, or expected profit.

$D_p$ or $D_p(\cdot)$ denote a demand function of $p$. We will consider three $D_p$–forms:

(a) the linear demand curve $D_{pl} = a - bp$, where $a$ and $b$ are positive constants, $p \leq a/b$;

(b) the iso-elastic curve $D_{pc} = Kp^{-\alpha}$, where $K$ and $\alpha$ are positive constants (we consider only cases with $\alpha > 1$, otherwise situations with infinite $\Pi_C$ would arise);

(c) a hybrid curve $D_{ph}$ that “mixes” a linear $D_{pl}$ with an iso-elastic $D_{pc}$, using parameter “$\tau$” as the mixture-ratio, as explained in LL (2003, 2005):

$$D_{ph} = \tau(a-bp)+(1-\tau)Kp^{-\alpha}, \hspace{1cm} (1)$$

Notice that $D_p$’s optional second subscript denotes the demand-function type: “l”, “c” or “h”.

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This paper considers the case of a dominant manufacturer (a “she”) selling to one retailer (a “he”). The manufacturer’s and the retailer’s deterministic profit functions are, respectively:

\[ \Pi_M = (w-c)V \quad \text{and} \quad \Pi_R = (p-w)V \] 

As justified in LL (2003, 2005), the typical “logistic cost components” (i.e., ordering and carrying costs) are ignored in (2) in order to reveal more effectively the differences between the traditional det-sym-i and the proposed sto-asy-i systems.

Formulas of optimal solutions for the one-echelon (integrated firm) are given in Table 1 (adapted from “Table 1” in LL 2005).

2.2 Brief Summary of Known Problematic Behavior of the Deterministic-Symmetric-Information (det-sym-i) Manufacturer-Stackelberg ([mS]) Framework

**Problematic Behavior 1 (“PB1”):** Retailer-Follower’s Profit Exceeds Manufacturer-Leader’s Profit Under an Iso-elastic \( D_{pc} \). E.g., in Table 2 of LL (2005), one sees in the “Iso-elastic \( D_{pc} \)” column that \( \Pi_M^* < \Pi_R^* \). Implications of this anomaly are discussed in LL (2005, §3.1 and §4). To resolve this anomaly, some earlier authors suggested that under an iso-elastic \( D_{pc} \) the dominant manufacturer should implement a retailer-Stackelberg game by making the retailer act as the “leader.” However, it is unclear exactly how this can be implemented, since the retailer knows that if he acts as a leader his profit will be less.

**Problematic Behavior 2 (“PB2”):** Instability of the Two-Echelon [mS] Process. LL (2003, §6, and 2005, §3) showed that, for some very plausible \( D_{pc} \) forms, a very minute change in the \( D_{pc} \)’s curvature can lead to a drastic change in the theoretically “correct” optimal decision for an [mS] game. In practice, this means that the dominant manufacturer will not know how to make a decision in such a situation.

We will show in §5 that our proposed sto-asy-i framework resolves both PB1 and PB2.

3. ANOTHER LOGICAL PROBLEM IN MANUFACTURER-STACKELBERG: WHEN A VOLUME DISCOUNT CAN BE OFFERED

Although the justification behind assuming a non-cooperative game such as “manufacturer Stackelberg” ([mS]) is that the players will not voluntarily cooperate, the literature using deterministic-and-symmetric-information (det-sym-i) models has accepted volume discounting as a legitimate channel coordination device. However, we show in this section that the volume-discount device reveals another inherent logical inconsistency of the det-sym-i [mS] framework. Incidentally, this paper adopts the convention used in, e.g., Viswanathan and Wang (2003) to differentiate between two types of quantity/volume
discounting: (i) “quantity” discount for a discount given on the basis of the amount ordered in each batch (regardless of the total amount per period); and (ii) “volume” discount for a discount given on the basis of the total amount ordered over a “period” (in this paper, this corresponds to the “period” over which $V$ is specified). Since our profit functions as stated in (2) do not contain logistics costs, only “volume” discounting is relevant in our model. This is somewhat fortunate because volume discounting is preferable when one takes into consideration the bullwhip effect.

The simplest and most widely taught volume discount format is the following: the wholesale price/unit is reduced from the regular level of $w$ to the discounted level of $w_d$ if the order volume equals or exceeds the “qualifying” volume $V_{dis}$. For a known linear $D_{pl} = a - bp$, consider now the following “degenerate” volume discount schedule (the $V_1^*$ and $p_i^*$ formulas for the linear-$D_p$ case are given in Table 1):

Define M as an arbitrarily large number satisfying $M > a/b$; then

\[
\begin{align*}
\text{(a)} & \quad \text{set } V_{dis} = V_1^*, \text{ and charge } w = M \text{ when } V < V_{dis} \\
\text{(b)} & \quad \text{charge } w_d = p_i^* - \varepsilon \text{ when } V \geq V_{dis} \quad \text{("$\varepsilon$" arbitrarily small)}
\end{align*}
\]

To illustrate, assume the following numerical parameters:

\[D_p = a - bp = 100 - 10p \quad \text{(i.e., } a = 100 \text{ and } b = 10); \quad c = 1\]

For these parameters, The Table-1 formulas give $p_i^* = 5.5$ and $V_1^* = 45$. Therefore, an example of a degenerate schedule according to (3) might be:

\[
\begin{align*}
\text{(a)} & \quad \text{charge } w = 10.1 \text{ when } V < 45; \\
\text{(b)} & \quad \text{charge } w_d = 5.45 \quad \text{(i.e., set } \varepsilon = 0.05) \text{ when } V \geq 45
\end{align*}
\]

Since $V = D_p(p) = 0$ if $p > a/b$ (i.e., if $p > 10$ in this case), one can easily see that the above schedule forces the retailer to coordinate perfectly; i.e., the optimal alternative for the retailer is to purchase $V_1^*$ units at $w_d$/unit and sell it at $p_i^$/unit. However, the retailer now makes an arbitrarily small profit of $\Pi_R = \varepsilon V_1^*$ — presumably a subsistence-level profit. That is, the manufacturer simultaneously attains a 100% C.E. and lionizes the channel’s profit — consistent with the [mS] attitude defined in §2.2. It can be easily shown that, for any other $D_{pr}$-form, the manufacturer can design similar degenerate discount schedules that not only attain a 100% C.E. but also empower her to allocate an arbitrarily-small subsistence $\Pi_R$ to the retailer. Therefore, under the det-sym-i framework, there is no reason why the (explicitly assumed) profit-maximizing and dominant manufacturer would not always implement this degenerate volume-discount scheme.
The preceding discussion shows that in a world where no discounting is “allowed,”
assuming an [mS] profit-maximizing “self-centered” manufacturer leads to a seemingly
meaningful problem and a “good-looking” standard textbook solution. Thus, at equilibrium
the retailer gets a reasonable share of the channel’s profit through his own “market power”
(whatever it might mean); e.g., under a linear $D_{pl}$ the retailer’s $\Pi^*_R$ is assured of one-third of
the channel’s $\Pi^*_C$ (see Table 2 in LL 2005). However, when discounting is allowed, if one
uses consistently the original [mS] assumption made in the no-discount situation, a
degenerate solution ensues — the retailer is unable to get any profit through his own “market
power.” Yet, the above outcome and the discounting scheme (i.e., (3)) that produces this
outcome are entirely consistent with the prevalent framework of a two-echelon [mS] game in
a det-sym-i environment.

In a det-sym-i system both players know exactly what it takes to maximize the channel’s
profit. Imposing an [mS] (or other kindred non-cooperative) game here means that the
players are: (i) on one hand sufficiently rational and smart to play the non-cooperative (say)
[mS] game under very definite rules; but (ii) on the other hand also sufficiently irrational to
opt for playing a non-cooperative game instead of simply “collaborating” and sharing the
maximized channel profit through negotiation. However, by setting $w$ at essentially a
“ridiculously high” value of “M”, the degenerate discounting scheme (3) is for all practical
purposes equivalent to the following manufacturer’s statement to the retailer: “If you want to
do business with me, you must buy exactly $V_1^*$ units, no more and no less, and at the unit
wholesale price of $(p_1^*-\varepsilon)$; take it or leave it.” As long as $\varepsilon$ is sufficiently large (as
presumably determined through negotiation), the retailer will submit. Thus, firstly, assuming
a game means assuming explicitly that the players would not simply coordinate and negotiate;
but then secondly, discounting is allowed by the gaming framework in the current literature,
and this discounting dimension in effect opens a backdoor way to return to the outright
“coordinate and negotiate” process which the gaming framework explicitly assumed away.

The above volume-discount degeneracy constitutes not only an additional example of the
logical circularity of assuming a det-sym-i non-cooperative game (pointed out in LL 2005); it
will also establish the desirability (as will be shown in §6 and §7) of using a sto-asy-i
framework when volume discounting is to be considered.

Note, incidentally, that the implicit assumption here is that the retailer wants to carry the
product and that no other manufacturer offers the identical product – which are the same
assumptions made in the standard “non-degenerate” discounting schemes.
4. THE STOCHASTIC-ASYMMETRIC-INFORMATION FRAMEWORK

4.1 Summary of the framework as presented in Lau and Lau (2005)

The sto-asy-i framework presented in LL (2005) assume a linear demand curve. At a “most likely” (or a recently implemented) retail price $p_o$, the “most likely” (or observed) demand is $D_o$. The rate of change in demand with retail price is known to be a constant, but the value of this constant rate is stochastic to the manufacturer at the time when she has to set $w$; i.e., the manufacturer perceives $D_p$ as:

$$D_{pl} = D_o - b(p - p_o), \text{ where } b \text{ is a random variable;}$$

(6)

or, equivalently,

$$D_{pl} = (D_o + \tilde{b} p_o) - \tilde{b} p = \tilde{a} - \tilde{b} p, \text{ where } \tilde{a} = D_o + \tilde{b} p_o$$

(7)

In contrast, the retailer knows the actual $b$-value, and the manufacturer is aware that the retailer has this knowledge. Thus, knowledge of market demand parameter $b$ is asymmetric between players and also stochastic to the manufacturer.

LL (2005) pointed out that, under their sto-asy-i framework, the dominant manufacturer may consider playing one the following three alternative “games” (listed in decreasing order of the level of channel coordination/integration): (i) Replicate the integrated firm (symbol [ri]); (ii) Enforce a manufacturer-imposed retail price $p_M$ before knowing the actual $\tilde{b}$-value (symbol [pm]); and (iii) Play the [mS] game. Table 5 in LL(2005) gives the formulas for the players’ profits for each of these three games.

4.2 Numerical Illustration of the Analytical Results Presented in LL (2005)

LL (2005) argued analytically that their Table-5 results generalize and rationalize the [mS] game. To better understand their claim and (more importantly) to demonstrate the necessity of the extensions to be presented in this paper, we present in Table 2 a numerical illustration of LL’s (2005) sto-asy-i framework using the following parameter values:

$$a = 100, \quad b = 10 \text{ (for } \tilde{D}_{pl} = \tilde{a} + \tilde{b}c), \quad \tilde{b} \text{ uniformly distributed,} \quad \sigma_b = b\kappa_b \text{ (} \kappa_b \text{ set to vary from 0 to 0.5);} \quad c = 1, \quad p_o = 3, \quad \text{and } D_o = 70.$$  

(8)

Since the range of a uniformly distributed $\tilde{b}$ is $\sigma_b \sqrt{12}$ and occurrence of a negative $b$-value must be avoided, the largest $\sigma_b$-value in Table 2 is 5.

First, recall that it was explained in §3 that it is unreasonable to assume that the players in a det-sym-i environment will not implement Alternative (i) — i.e., [ri], the highest level of coordination. In contrast, as explained in §7.2 of LL (2005), the sto-asy-i framework provides a genuine reason (namely, “impracticality”) for not implementing [ri].
Second, when stochasticity (namely, \( \hat{b} \)’s uncertainty) is still sufficiently low, the dominant manufacturer should implement Alternative (\( ii \)) (i.e., [pm], the medium level of coordination). Thus, Table 2 shows that when \( \kappa_b \) (see column 1) is small, [pm]’s \( \Theta_c^* \) (see column 4) exceeds [mS]’s \( \Theta_c^* \) (see column 7).

Third, when stochasticity becomes sufficiently high, it becomes worthwhile for the dominant manufacturer to play [mS] — i.e., operate at the lowest level of coordination. In Table 2, when \( \kappa_b \) (column 1) rises to around 0.47, [mS]’s \( \Theta_c^* \) (column 7) overtakes [pm]’s \( \Theta_c^* \) (column 4). Note also that at the same time [mS]’s \( \Theta_R^* \) (column 6) overtakes [mS]’s \( \Theta_M^* \) (column 5). The fact that the retailer’s \( \Theta_R^* \) (column 6) increases with \( \sigma_b \) follows the principle that a retailer earns his keep by dealing with retail-market’s uncertainty (explained in, e.g. Lau & Lau 1999);

Thus, the dominant manufacturer is not restricted to playing only one game (i.e., [mS]) in our sto-asym-i framework. Instead, she has three perfectly legal and practical choices: (i) play [ri] when \( \sigma_b \) is zero; (ii) play [pm] when \( \sigma_b \) is non-zero but small; (iii) play [mS] when \( \sigma_b \) is sufficiently large. Choice (iii) also illustrates an oft-observed phenomenon: the advantage of tighter coordination decreases as uncertainty increases.

However, the analytical results of LL’s (2005) Table 5, as numerically illustrated here in Table 2, pertain only to a linear \( D_{pl} \), hence they do not address the flaws pointed out in §2.2; i.e., flaws revealed when non-linear \( D_{pl} \)s are used. The single-\( w \) scheme considered in Table 2 also cannot address the logical flaw pointed out in §3 when discount schemes are considered. Section 5 will present results of the sto-asym-i framework under an iso-elastic \( D_{pc} \), while §6 and §7 will present results on how volume discount schemes might be designed under the sto-asym-i framework. The results in §5 to §7 demonstrate the superiority of the sto-asym-i framework by showing how it can resolve the difficulties explained in §2.2 and §3 for the det-sym-i framework.

5. THE STOCHASTIC-ASYMMETRIC-INFORMATION FRAMEWORK UNDER AN ISO-ELASTIC CURVE

5.1 Analytical Developments

In order to address problematic behavior PB1 as stated in §2.2, it is necessary to consider an iso-elastic \( D_{pc} \). However, LL (2005) considered only a linear \( D_{pl} \). Unfortunately, when \( D_p \) is iso-elastic, it is not possible anymore to obtain closed-form profit expressions such as those shown in LL’s (2005) Table 5. We sketch below the procedure for determining the players’ optimal profit for an [mS] game under \( D_{pc} \). Procedures for determining the optimal
profits for the [ri] and [pm] processes are similar but considerably simpler. The remainder of this subsection (§5.1) can be skipped without loss of continuity.

Using the same approach for expressing the linear $\bar{D}_{pl}$ in (7), the stochastic iso-elastic demand function is now $\bar{D}_{pc} = D_o(p_o/p)^{\hat{\alpha}}$. Note that an exponential expression ($x^k$) is written as $(x^{\hat{k}})$ here so that the random-variable symbol “~” of the exponent “$\hat{k}$” can be clearly shown. Under an [mS] game, without knowing the actual $\hat{\alpha}$-value, the manufacturer sets $w$. She keeps in mind that for any given $w$ and the actual $\hat{\alpha}$-value known to the retailer, he will set a $p$-value that maximizes $\bar{\Theta}_R$, which is (adapting Table-1’s $p_l^*$-formula):

$$\tilde{p}_w = \hat{\alpha}w/(\hat{\alpha}-1)$$  \hfill (9)

Substituting the above $\tilde{p}_w$ into $\bar{D}_{pc}$ gives the corresponding sales volume as

$$\bar{V}_w = D_o \cdot \left[ \frac{p_o(\hat{\alpha}-1)/(w \hat{\alpha})} {w^{\hat{\alpha}}} \right]^{\hat{\alpha}}.$$  \hfill (10)

Therefore, the profit of an [mS]-implementing manufacturer is:

$$\bar{\Theta}_M = (w-c) \bar{V}_w = (w-c)D_o \left[ \frac{p_o(\hat{\alpha}-1)} {w^{\hat{\alpha}}} \right]^{\hat{\alpha}}.$$  \hfill (11)

The manufacturer’s problem is to determine the value of $w^*$ that maximizes $E(\bar{\Theta}_M)$, or $\bar{\Theta}_M^*$. Thus, under [mS], the manufacturer’s problem can be stated as:

$$\text{Find } w^* \text{ that maximizes } \int_{-\infty}^{\infty} (w-c)D_o \left[ \frac{p_o(x-1)} {wx} \right]^x f_\alpha(x)dx,$$  \hfill (12)

where $f_\alpha(x)$ is the density function of $\bar{D}_{pc}$’s exponent $\hat{\alpha}$. Although (12) does not have a closed-form analytical solution, it can be easily solved numerically. In this study the IMSL (1994) library of FORTRAN subroutines are used to perform the integrations and optimizations needed to solve (12) and other similar problems. After $w^*$ is determined, $\bar{\Theta}_R^*$ can be computed by substituting $w^*$ into (9), (10) and the relationship “$\bar{\Theta}_R = \bar{V}(\tilde{p} - w)$.”

Consider now the [pm] arrangement. Combining the approach explained above with the approach explained in LL (2005) for deriving the $\Theta_C$-expression in “alternative (ii)” (see LL (2005)’s Table 5), it can be easily shown that the problem under [pm] can be stated as:

$$\text{Find } p_M \text{ that maximizes } \Theta_C = (p_M-c)\int_{-\infty}^{\infty} D_o(p_o/p_M)^x f_\alpha(x)dx.$$  \hfill (13)

Thus, the following needs to be emphasized: Under a stochastic linear $\bar{D}_{pl}$, referring to Table-1’s $p_l^*$-formula gives $p_M$ as simply $(a+bc)/(2b)$ — this was stated as expression (26) in LL (2003). In contrast, under a stochastic iso-elastic $\bar{D}_{pc}$, $p_M^*$ can only be determined numerically through solving (13).
5.2 Numerical Examples and the Resolution of Problematic Behavior

Table 3 presents the relevant answers for the following numerical example:

Iso-elastic demand curve $\hat{D}_{pc} = D_o(p_o/p)^{(\hat{a})}$, where $\hat{a}$ is uniformly distributed with mean of $\alpha = 2.5$. Also, $p_o = 3$, $c = 1$, $D_o = 15$. \(14\)

Given $\alpha = 2.5$, since (i) situations with $\alpha < 1$ produce infinite $\Pi_C$ and must be avoided; and (ii) the range of $\alpha$ is $\sigma_{\alpha} = \sqrt(12)$, we only consider $\sigma_{\alpha}$-values up to 0.8 so that the realized $\hat{a}$-values will always exceed 1.

Comparing Table 3 with Table 2, notable observations are:

(i) As in Table 2, the following Table-3 values increase as $\sigma_{\alpha}$ increases: $\Theta_1^*$ (column 2), $\Theta_R^*$ (column 6). However, the values of $\Theta_C^*$ (column 3) and $\Theta_M^*$ (column 5) also increase with $\sigma_{\alpha}$ in Table 3, whereas these values do not change with $\sigma_b$ in Table 2 (columns 4 and 5).

(ii) In both Tables 2 and 3, as $\sigma_{\alpha}$ (or $\sigma_b$) increases, one observes a decrement in the ratio between $\Theta_C^*$ and $\Theta_M^*$; i.e., (column 4 ÷ column 7) in Table 2 and (column 3 ÷ column 7 → column 8) in Table 3. This follows the principle that the advantage of “tighter” coordination decreases as uncertainty increases, as stated earlier in §4.2. However, in contrast to Table 2, $\Theta_C^*$ never falls below $\Theta_M^*$ in Table 3. This simply means that the manufacturer should choose to implement $\Theta_M$ for all levels of $\alpha$-uncertainty.

Consider now PB1 stated in §2.2. First, Table 3 shows that when $D_p$ is iso-elastic, $\Theta_M^*$ (column 5) is still always smaller than $\Theta_R^*$ (column 6) in our sto-asy-i model; i.e., we are still stuck with the phenomenon that the dominant manufacturer-leader makes less than the retailer-follower (the bothersome characteristic stated in §2.2.1). However, this characteristic does not pose a problem anymore under our sto-asy-i framework. This is because, as explained in §4, even under a linear $\hat{D}_p$, the dominant manufacturer should only switch from $\Theta_M$ to $\Theta_R$’s uncertainty (or $\hat{b}$’s uncertainty) is sufficiently high. Table 3 simply depicts a situation where the dominant manufacturer should always implement $\Theta_M$. Since the manufacturer is explicitly assumed as “dominant” in the classical [mS] framework, such a dominant manufacturer is allowed to implement $\Theta_M$ if she wishes to (see Appendix 1 for additional justifications). In contrast, it is much less obvious how she can actually implement the alternative of “making the retailer act as the leader when the demand curve is iso-elastic” (as implied in the current literature). When implementing $\Theta_M$, column 4 of 6 shows that under an iso-elastic $D_{pc}$ the C.E. remains very high as uncertainty
(σα) increases, but columns 3 and 4 of Table 2 indicate that the same cannot be said for a linear $D_{pl}$ — another advantage of using [pm] under $D_{pc}$. Incidentally, one might note that after one recognizes the advantage of [pm] by looking at the row “σα = 0,” its use in resolving **PB1** can be invoked independent of our sto-asy-i framework. However, the sto-asy-i framework covers a wider perspective that includes the linear $D_{pl}$ as well as the stochastic forms of $D_{pc}$ and $D_{pl}$.

The “instability” problem stated as **PB2** in §2.2 can also be resolved by using [pm]. Table 3 in LL (2005) shows that, while the values of $w_M^*$ and $p_R^*$ are extremely sensitive to the very small differences between the iso-elastic $D_{pc}$ and the hybrid $D_{ph}$ considered in their illustration, the values of $p_l^*$ remain very stable. Thus, faced with the uncertainty that the market’s demand curve may be either $D_{pc}$ or $D_{ph}$, the dominant manufacturer should simply implement [pm] instead of [mS] and achieve much better results not only for herself but also for the channel. Recall that the $D_{ph}$ illustrated in Table 3 of LL (2005) is primarily an iso-elastic $D_{pc}$ slightly contaminated by a linear-$D_{pl}$ component; thus, had the manufacturer not been told that the actual demand curve may be $D_{ph}$ instead of $D_{pc}$, she would have implemented [pm] as explained in the preceding paragraph. One might also perceive the current scenario, where the actual $D_p$ may be either $D_{pc}$ or $D_{ph}$, as another form of demand stochasticity.

Although the preceding discussions are based on the numerical answers for the set of parameters specified in (14), we need to emphasize the following:

(i) A grid system has been used to generate many different combinations of parameter values, and sets of numerical answers corresponding to those presented in Table 3 have been examined to ensure that our preceding discussions are valid for other combinations of parameter values.

(ii) The same approach is used in the following sections when patterns of numerical solutions are discussed.

Since [pm] has turned out to be a very desirable arrangement, we briefly discuss its legal aspect in Appendix 1.

6. **A STOCHASTIC AND ASYMMETRIC-INFORMATION MANUFACTURER-STACKELBERG GAME WITH VOLUME DISCOUNT, LINEAR $\hat{D}_{pl}$**

6.1 **Statement of the Problem**

We will consider the simplest and most widely taught volume discount scheme, defined by the 3-tupel ($w, w_d, Q_d$); i.e., the wholesale price/unit is reduced from the regular level of $w$.
to the discounted level of $w_d$ if the (periodic, or annual) order volume $V$ equals or exceeds the “qualifying” volume $Q_d$. This section considers the case of a linear $D_{pl} = D_o - b(p - p_o) = \tilde{a} - \tilde{b}p$; recall also that $\tilde{b}$ is assumed to be known to the manufacturer only as a random variable, but the retailer knows the actual $b$-value (and hence the $a$-value). We show in (A16) and (A17) of Appendix 2 that, for any given set $(w, w_d, Q_d)$-values, the expected profit of the manufacturer in a [mS] game is:

$$\Theta_M = \frac{a -bw}{2}(w-c) + \int_{L_3} \left[ \frac{D_o + x(p_o - w)}{2} (w - c) \right] f_{\tilde{b}}(x)dx$$

$$+ (w_d - w) \int_{L_5} \frac{D_o + x(p_o - w - w_d + c)}{2} f_{\tilde{b}}(x)dx$$

for $p_o \geq w_d$ \hspace{1cm} (15a)

$$\Theta_M = \frac{a -bw}{2}(w-c) + \int_{L_3} \left[ \frac{D_o + x(p_o - w)}{2} (w - c) \right] f_{\tilde{b}}(x)dx$$

$$+ (w_d - w) \int_{L_5} \frac{D_o + x(p_o - w - w_d + c)}{2} f_{\tilde{b}}(x)dx$$

for $p_o < w_d$ \hspace{1cm} (15b)

where the integral limits $L_1, L_3$ and $L_5$ are defined in (A11) to (A13) in Appendix 2.

6.2 Numerical Results for a Manufacturer-Stackelberg Game

The manufacturer’s problem of finding the optimal volume discount schedule $(w, w_d, Q_d)^*$ that maximizes $\Theta_M$ in (15) is a straightforward nonlinear programming problem with three decision variables and can be easily solved numerically. To illustrate, we will assume the same parameter values defined in (8) for constructing Table 2, i.e., $a = 100, \ b = 10, \ \tilde{b}$ uniformly distributed, $c = 1, \ p_o = 3, \ D_o = 70$. Consider the case with $\sigma_b = 2$. The discount schedule that maximizes $\Theta_M$ in (15) is found to be:

Regular wholesale price $w^* = 8.186$ if order volume is less than $Q_d^* = 48.545$;

Discount wholesale price $w_d^* = 4.593$ if order volume is not less than $Q_d^* = 48.545$.

With this discount schedule the expected profits of the manufacturer’s and the retailer’s are, respectively, 174.45 and 31.29.

Table 4 depicts the effect of $b$’s uncertainty (i.e., $\sigma_b$) on the optimal solutions; its “$\sigma_b = 2$” row corresponds to the solution in the preceding paragraph. Note that the $\Theta_I^*$ values here are the same as the Table-2 column-3 $\Theta_I^*$ values, hence they are not repeated in Table 4. The “$\sigma_b = 0$” row repeats the conclusion reached in §3 and highlights the fact that the deterministic-\$b scenario is a special case of our generalized sto-asy-i model. However, when $\sigma_b$ is non-zero the manufacturer can no longer design a $(w, w_d, Q_d)$-type volume discount schedule that gives herself an arbitrarily large share of the channel’s profit. Also, as $\sigma_b$ increases, the retailer’s
profit share on the basis of his own “market power” (i.e., $\Theta_R^*$ in column #7) increases, which
definitely merely echoes the principle reviewed earlier in §4.2: “a retailer earns his keep by
dealing with the retail-market’s uncertainty.” Note that when $\kappa_i$ reaches 0.5, the column-8
ratio ($\Theta_M^*/\Theta_R^*$) is 1.68; i.e., it actually drops below the ($\Theta_M^*/\Theta_R^*$) of a deterministic [mS] process — which is 2 (see Table 2 in LL 2005).

Thus, under our sto-asy-i framework, when a [mS] manufacturer designs a volume
discount scheme to maximize her profit (consistent with [mS]’s basic premises); a degenerate
solution will not ensue — in contrast to the det-sym-i framework explained in §3.

We summarize what we have shown so far:

(A) When volume discounting is NOT to be used (for whatever reason), the dominant
manufacturer in a sto-asy-i environment may implement either [mS] or [pm];
(B) Given the volume-discount option, the dominant manufacturer should choose to play an
[mS] game that is coupled with a volume discount scheme.

Point (B) relates partially to the currently well-known notion of the advantage of volume
discount. However, in the existing det-sym-i literature the volume discount can be
introduced only with the simultaneous alteration of the dominant manufacturer’s objective
(i.e., she does not maximize her own profit anymore). With our sto-asy-i framework the
original objective need not be altered.

Another important characteristic depicted in Table 4 is the very high C.E.s (compared to
the C.E.s shown in Table 2 for the no-discount-[mS] counterpart). It is of course already well
known that a volume discount scheme can attain a 100% C.E. in the det-sym-i case. Table 4
now shows that even when the manufacturer faces considerable uncertainty in $\tilde{b}$, she can still
achieve a close-to-100% C.E. by designing a proper ($w, w_d, Q_d$) schedule.

Table 5 summarizes the channel’s and the players’ expected profits under the three viable
sto-asy-i processes considered so far; namely, the [pm] game and the “[mS] without discount”
game considered in §4 and Table 2, plus the “[mS] with discount” game considered in this
section. The three $\Theta_C^*$ columns under “Total Channel Profit $\Theta_C^*$” in Table 5 show that (as
expected intuitively), the “[mS] with discount” process gives consistently the highest $\Theta_C^*$.

Comparing Tables 2 and 4 also reveals an important function of volume discounting
hitherto overlooked in the academic literature, because this function is only revealed under a
sto-asy-i framework. The function is: volume discounting serves as a mechanism for the
manufacturer to benefit from the retailer’s $b$-knowledge, even though the manufacturer does
not share the knowledge itself. Note that in a det-sym-i context, the manufacturer benefits
from a volume discount scheme by merely forcing the retailer to operate at a single desired level — the channel-optimal $V_I^*$ level (which is deterministically and a-priori known to both players). In contrast, in a sto-asy-i context the same simple volume discount format ($w, w_d, Q_d$) does not force the retailer to operate at a pre-determined level. The “ignorant” manufacturer lets the retailer make the right decision with his $b$ knowledge (this mechanism can be better appreciated by following $\Theta_M^*$’s mathematical derivation given in Appendix 2).

If the actual-$b$ value (known to the retailer) is relatively low (“Situation 1”, defined clearer in Appendix 2), he will opt to take advantage of the discount and in effect move the channel towards a higher volume of operation (by ordering at least $Q_d$). However, if the known $b$-value is relatively high (“Situation 2,” defined clearer in Appendix 2), he will opt to forgo the discount (and pay $w/unit) and thus move the channel towards a lower volume of operation. Moreover, even within each of the two “Situations”, the volume of operation (i.e., the “$V$” value) is still not single-valued; it varies with the retailer’s observed $b$-value. Tables 2 and 4 show that volume discounting increases $\Theta_M^*$ not only through cannibalizing the retailer’s $\Theta_R^*$ (i.e., Table 4’s column-7 entries are less than the Table 2’s column-6 entries), but also through enlarging $\Theta_C^*$ (i.e., through the retailer’s use of his $b$-knowledge in ordering).

Closely related to the mechanism explained above is the observation that a volume-discount schedule’s $w$ needs to be “conscientiously” set only under the sto-asy-i framework. Referring to §3 and (3), it can be seen that under the det-sym-i framework, $w$ can be arbitrarily set at any value above $a/b$, because $w$ is not meant to be actually used.

### 6.3 A “Better Looking” Volume Discount Scheme

Noting that “$1-(w_d/w)$” gives the discount rate under a discount scheme, the Table-4 volume discount schemes (for the non-degenerate cases with $\sigma_b \neq 0$) call for a discount of more than 40%. A manufacturer quoting such a large discount may either appear to be somewhat capricious or be betraying her large profit margins. Assume that the manufacturer wants to offer only a (say) 20% volume discount. Imposing this constraint actually leads to a simpler mathematical problem, because in maximizing the $\Theta_M$-functions given in (15), the number of free decision variables is now reduced from 3 in ($w, w_d, Q_d$) to 2. The answers (for the same parameters given in (8)) are given in Table 6. Understandably, the $\Theta_M^*$-values in Table 6 are less than their $\Theta_M^*$-counterparts in Table 4, however, they are still consistently higher than the no-discount $\Theta_M^*$-counterparts given in Table 2. As for the C.E., the C.E. figures in Table 6 may or may not be higher than their counterparts in Table 4, but they are consistently higher than their no-discount C.E.-counterparts given in Table 2. The retailer is
the major beneficiary — the $\Theta_R$’s in Table 6 are consistently higher than their counterparts in Table 4. Theoretically, one recognizes that imposing an additional constraint (in this case a discount-magnitude constraint) can never benefit the optimizer (i.e., the manufacturer); nevertheless, the preceding finding may be perceived in a form that appears counter-intuitive – i.e., the retailer benefits (and the manufacturer loses) when the manufacturer is restricted from offering too high a discount rate to her retailer!

Table 6 depicts a relatively more plausible world. First, when one moves from a manufacturer oblivious to the “volume discount” tool to one that recognizes that this tool is at her disposal, there is no need to simultaneously assume the sudden conversion of this manufacturer from a selfish [mS] profit maximizer to a caring player concerned about her opponent’s welfare in order to avoid “degeneracy” (explained in §3). Second, it is not necessary for the manufacturer to offer suspicion-arousing discount rates depicted in Table 4. Table 6 depicts schemes with realistic discount rates; both players retain their original and realistic [mS] objective and are able to obtain plausible shares of the channel profit.

Furthermore, the C.E.s are very respectable and significantly higher than the 75% level for the det-sym-i counterpart (see Table 2 in LL 2005).

This section demonstrated the merits of our sto-asy-i framework when one begins to consider the manufacturer’s volume-discounting option. However, while the examples in this section agree with the current literature’s notion that a manufacturer can design a volume discount scheme $(w, w_d, Q_d)$ that performs better than a single-price scheme, we will show in the following §7 that this notion cannot be safely generalized to situations where the demand curve is non-linear – a phenomenon that further illustrates the merit of our sto-asy-i framework.

### 7. A STOCHASTIC AND ASYMMETRIC-INFORMATION MANUFACTURER-STACKELBERG GAME WITH VOLUME DISCOUNT, ISO-ELASTIC $D(p)$

We again consider the simple volume discount scheme defined by the 3-tupel $(w, w_d, Q_d)$. Equations (A26) in Appendix 3 shows that, for any given set of $(w, w_d, Q_d)$-values, the expected profit of the manufacturer in a [mS] game is:

**Condition 1:** when $p_o > w_d$, 

$$
\Theta_M = \int_{L_0}^{+\infty} \Theta_{M1} f_o(x)dx + \int_{L_1}^{L_0} \Theta_{MX} f_o(x)dx + \int_{L_2}^{L_1} \Theta_{MY} f_o(x)dx \quad \text{for } p_o > w \quad (16.1)
$$

$$
\Theta_M = \int_{L_0}^{+\infty} \Theta_{M1} f_o(x)dx + \int_{L_1}^{L_0} \Theta_{MX} f_o(x)dx + \int_{L_2}^{L_1} \Theta_{MY} f_o(x)dx \quad \text{for } w_d < p_o \leq w \quad (16.2)
$$
Condition 2: when \( p_o = w_d \)

\[
\Theta_M = \int_{-\infty}^{+\infty} \Theta_{MX} f_\alpha(x) \, dx \quad \text{for } D_o/Q_d < e \quad (e \equiv \text{natural constant}) \quad (16.3)
\]

\[
\Theta_M = \int_{-\infty}^{+\infty} \Theta_{M1} f_\alpha(x) \, dx + \int_{1}^{t_\alpha} \Theta_{MX} f_\alpha(x) \, dx \quad \text{for } D_o/Q_d \geq e \quad (16.4)
\]

Condition 3: when \( p_o < w_d \)

\[
\Theta_M = \int_{1}^{+\infty} \Theta_{MX} f_\alpha(x) \, dx \quad \text{for } D_o/Q_d < h(\alpha_1) \quad (16.5)
\]

\[
\Theta_M = \int_{t_\alpha}^{+\infty} \Theta_{M1} f_\alpha(x) \, dx + \int_{1}^{t_\alpha} \Theta_{MX} f_\alpha(x) \, dx + \int_{1}^{+\infty} \Theta_{MY} f_\alpha(x) \, dx \quad \text{for } D_o/Q_d \geq h(\alpha_1) \quad (16.6)
\]

See Appendix 3 for the definitions of the integrand-components \( \Theta_{M1}, \Theta_{MX} \) and \( \Theta_{MY} \); the quantity \( h(\alpha_1) \) as well as the integral limits \( \alpha_2, \alpha_3 \) and \( L_i \, (i = 0 \text{ to } 4) \) used in (16).

The manufacturer’s problem is to find the set of optimal set of \( (w, w_d, Q_d) \)-values that maximize \( \Theta_M \). As in §5, this problem can be solved numerically. To illustrate, we will assume that the elasticity parameter \( \tilde{\alpha} \) in \( K^\alpha \) is uniformly distributed with a mean of \( \alpha = 2.5 \) and \( \sigma_\alpha \) ranging from 0.1 to 0.8 (the same parameter values used in Table 3 of §5). The answers are presented in Table 7. Perhaps the most eye-catching characteristic is that now \( \Theta_M^* \) is consistently larger than \( \Theta_R^* \); i.e., even when \( D(p) \) is iso-elastic, the dominant manufacturer is now able to get a larger profit share than the retailer without resorting to the dubious mechanism of somehow inducing the retailer to be the leader. However, scrutinizing the numbers in Tables 3 and 7 reveals something much more important — i.e., the unexpected attractiveness of the \([pm]\) arrangement when the demand curve is iso-elastic. To elaborate:

(i) As summarized in §4.1, LL (2005) pointed out that under a sto-asy-i system it is impractical to replicate an integrated firm (even if the players would like to do it) and achieve a 100% C.E..

(ii) Under a linear \( \tilde{D}_{pl} \), \([pm]\)’s \( \Theta^*_C = 202.5 \), at column 4 of Table 2) is always less than the \( \Theta^*_C \) in Table 4 (column 9); it is also less than the \( \Theta^*_C \) in Table 6 when \( \kappa_b \) is sufficiently high. In other words, under \( \tilde{D}_{pl} \), volume discount continues to work better than \([pm]\) in most situations. In contrast, if the \( \tilde{D}_p \) is iso-elastic, then for all \( \kappa_p \)-values, \([pm]\)’s \( \Theta^*_C \) given in column 3 of Table 3 is always higher than the corresponding \( \Theta^*_C \) of the optimal volume-discount scheme given in Table 7. Thus, while a discounting is superior to the “plain” (single-w) \([mS]\) under both \( \tilde{D}_{pl} \) and \( \tilde{D}_{pc} \), discounting is superior to \([pm]\) only under \( \tilde{D}_{pl} \) but NOT under \( \tilde{D}_{pc} \).
This phenomenon addresses the question raised at the end of §6. Note that volume
discounting is the most widely studied channel-coordinating and manufacturer-benefiting
device in the literature. The preceding sections have suggested [pm] as a viable alternative
channel-coordinating device, and this section shows that, under an iso-elastic $D_{pc}$, a single-
price scheme (albeit using $p_M$ under [pm] instead of a $w^*$ under [mS]) can be a considerably
better channel-coordinating device than volume discounting.

8. SUMMARY AND CONCLUSION

8.1 Summary

We considered a dominant manufacturer wholesaling a product to a retailer, who in turn
retails it at $p$/unit to the consumers. The retail-market demand volume $V$ varies with $p$
according to a given demand curve $D_p$. A very common approach is to assume that the
“players” will play a manufacturer-Stackelberg ([mS]) game under a “deterministic and
symmetric-information” (“det-sym-i”) framework. In §2 and §3 we explained the logical
flaws of this approach, which are: (i) the dominant manufacturer-leader will have a lower
profit than the retailer under an iso-elastic $D_{pc}$; (ii) in some situations the system’s “correct
solution” can be hyper-sensitive to minute changes in the demand curve; (iii) applying
volume discounting while keeping the original [mS] profit-maximizing objective leads to an
implausible degenerate solution in which the manufacturer has dictatorial power over the
channel. This paper extends the “stochastic and asymmetric-information” (“sto-asy-i”) framework
proposed in Lau and Lau (2005), coupled with the notion that a profit-maximizing
dominant manufacturer may implement not only [mS] but also “[pm]” (i.e., using a
manufacturer-imposed maximum retail price). We show that this new framework resolves all
the logical flaws stated above. Specifically:

(i) The dominant manufacturer’s problematic inferior profit under an iso-elastic $D_{pc}$ (i.e.,
**PB1**) is resolved by recognizing that she should implement [pm] instead of [mS] under $D_{pc}$;
in contrast, recall that she should implement [mS] instead of [pm] under a linear $D_{pl}$ with high
parameter uncertainty.

(ii) Implementing [pm] also resolves the hyper-sensitive problem that may arise with some
non-linear demand curves, because in those situations a [pm] solution is not only stable but
also more profitable to the dominant player.

(iii) Under the sto-asy-i environment, if $D_p$ is linear, the optimal [mS]-discounting scheme
available to a dominant manufacturer will no longer be degenerate. However, if $D_p$ is iso-
elastic, the dominant manufacturer should use [pm] instead of a discounting scheme.
Using our sto-asy-i framework to resolve the logical flaws of the det-sym-i framework revealed two noteworthy points: (i) the attractiveness of the perfectly legal but overlooked channel-coordination mechanism [pm]; and (ii) volume discounting as a means for the dominant manufacturer to benefit from information known only to the retailer.

Along the way, we also presented a procedure for the dominant manufacturer to design a profit-maximizing volume discount scheme.

8.2 Future Extensions

A large number of studies have considered two-echelon supply chains with asymmetric information; see, e.g., Ha (2001), Corbett et al. (2004) and their references. However, many of these models consider information asymmetry with respect to either logistic-cost-component parameters or the retailer’s variable handling costs; these components are explicitly excluded in our profit functions stated in (2) because we wanted to concentrate on the effects of stochasticity and asymmetry in the demand curve. The effects of including logistic/variable costs and their information asymmetry into our model remain to be studied.

In §6 and §7 we have considered only the simplest discounting format. Our results in §7 indicate that, if \( D_p \) is iso-elastic, then [pm] is superior to this simple discount scheme for a dominant manufacturer. One might investigate whether a more sophisticated discounting scheme or other channel-coordinating devices may surpass [pm].

As shown in Appendix 1, although our legitimate maximum-price-maintenance [pm] does occur in the real world, it is admittedly rare. In contrast, the illegal minimum-price-maintenance variation of [pm] is much more well-known. The practical issues of implementing maximum-price-maintenance needs to be examined. In the past decades, many new channel-coordinating devices have been proposed and/or used by a small number of firms; among them are “revenue sharing” (see, e.g., Pasternack 2002) and “quantity-flexibility contracts” (see, e.g., Tsay 1999).

Section 7 shows that the relative merits of [pm] versus discounting is reversed when one switches from \( \tilde{D}_{pl} \) to \( \tilde{D}_{pc} \); it is unclear what their relative merits would be under other \( \tilde{D}_p \) functional forms. The need to consider the effect of different \( \tilde{D}_p \)-assumption has of course been raised by LL (2003).
TABLE 1: Summary of One-Echelon (Integrated Firm) Formulas

<table>
<thead>
<tr>
<th></th>
<th>Linear $D_{pl} = a-bp; \ c&lt;(a/b)$</th>
<th>Iso-elastic $D_{pc} = K \ p^{-\alpha}; \ \alpha&gt;1$</th>
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<td>$p^*_1$</td>
<td>$(a+bc)/(2b)$</td>
<td>$\alpha/(\alpha-1)$</td>
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<td>$I^*_1$</td>
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<td>$K(\alpha-1)^{-1}/(\alpha c^{\alpha-1})$</td>
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TABLE 2: Optimal Profits in the [ri], [pm] and [mS] Processes; Linear $D(p)$

$\Theta$ denotes profit, the subscripts I, C, M and R stand for, respectively, “integrated firm,” “channel,” “manufacturer” and “retailer.”

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<td>Retailer’s $\Theta^*_R$</td>
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TABLE 3: Optimal Profits in the [ri], [pm] and [mS] Processes; Iso-elastic $D(p)$

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<td>50.62</td>
<td>47.74</td>
<td>0.943</td>
<td>13.45</td>
<td>24.97</td>
<td>38.42</td>
<td>1.24</td>
</tr>
<tr>
<td>0.8</td>
<td>53.17</td>
<td>49.29</td>
<td>0.927</td>
<td>13.92</td>
<td>26.70</td>
<td>40.62</td>
<td>1.21</td>
</tr>
</tbody>
</table>
TABLE 4: Effects of $\sigma_b$ on the Optimal Solutions; $\Theta^*_M$–Maximizing Volume-Discounting Manufacturer; Linear $D(p)$

<table>
<thead>
<tr>
<th>$\kappa_b$</th>
<th>$\sigma_b$</th>
<th>$w^*$</th>
<th>$w'_d$</th>
<th>$Q_d^*$</th>
<th>$\Theta_M^*$</th>
<th>$\Theta_R^*$</th>
<th>$\Theta_M^<em>/\Theta_R^</em>$</th>
<th>$\Theta_C^*$</th>
<th>C.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>&gt;10</td>
<td>5.5-$\epsilon$</td>
<td>45</td>
<td>202.5 - 45 $\epsilon$</td>
<td>Arbitrarily large</td>
<td>202.50</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>8.917</td>
<td>4.982</td>
<td>46.75</td>
<td>186.15</td>
<td>17.18</td>
<td>10.83</td>
<td>203.33</td>
<td>0.998</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
<td>8.186</td>
<td>4.593</td>
<td>48.55</td>
<td>174.45</td>
<td>31.29</td>
<td>5.57</td>
<td>205.74</td>
<td>0.990</td>
</tr>
<tr>
<td>0.3</td>
<td>3</td>
<td>7.591</td>
<td>4.303</td>
<td>50.20</td>
<td>165.81</td>
<td>44.74</td>
<td>3.71</td>
<td>210.55</td>
<td>0.976</td>
</tr>
<tr>
<td>0.4</td>
<td>4</td>
<td>7.220</td>
<td>4.015</td>
<td>52.20</td>
<td>160.09</td>
<td>59.03</td>
<td>2.71</td>
<td>219.12</td>
<td>0.949</td>
</tr>
<tr>
<td>0.5</td>
<td>5</td>
<td>7.195</td>
<td>3.811</td>
<td>54.56</td>
<td>158.04</td>
<td>94.28</td>
<td>1.68</td>
<td>252.32</td>
<td>0.948</td>
</tr>
</tbody>
</table>

$^1\Theta_I^*$ same as in Table 2.

$^2\epsilon$ can be arbitrarily small.

TABLE 5: Comparison of $\Theta^*_C$, $\Theta^*_M$ and $\Theta^*_R$ in $[\text{pm}]$, $[\text{mS}]$ without Discount and $[\text{mS}]$ with Discount; Linear $D(p)$

<table>
<thead>
<tr>
<th>$\sigma_b$</th>
<th>$\Theta_C^*$</th>
<th>$\Theta_M^*$</th>
<th>$\Theta_R^*$</th>
<th>$\Theta_M^* + \Theta_R^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>202.50</td>
<td>151.875</td>
<td>50.625</td>
<td>202.50</td>
</tr>
<tr>
<td>1</td>
<td>202.50</td>
<td>153.12</td>
<td>51.87</td>
<td>203.33</td>
</tr>
<tr>
<td>2</td>
<td>202.50</td>
<td>157.16</td>
<td>55.91</td>
<td>205.74</td>
</tr>
<tr>
<td>3</td>
<td>202.50</td>
<td>165.12</td>
<td>63.87</td>
<td>210.55</td>
</tr>
<tr>
<td>4</td>
<td>202.50</td>
<td>180.26</td>
<td>79.01</td>
<td>219.12</td>
</tr>
<tr>
<td>5</td>
<td>202.50</td>
<td>215.66</td>
<td>114.41</td>
<td>252.32</td>
</tr>
</tbody>
</table>

$^1\Theta_M^*$ and $\Theta_R^*$ under $[\text{pm}]$ are negotiated (see Table 5 in LL (2005)), hence they are not listed.

TABLE 6: Effects of $\sigma_b$ on the Optimal Solutions; $\Theta^*_M$–Maximizing Manufacturer, 20% Discount Permitted, Linear $D(p)$

<table>
<thead>
<tr>
<th>$\sigma_b$</th>
<th>$w^*$</th>
<th>$w'_d$</th>
<th>$Q_d^*$</th>
<th>$\Theta_M^*$</th>
<th>$\Theta_R^*$</th>
<th>$\Theta_M^<em>/\Theta_R^</em>$</th>
<th>$\Theta_C^*$</th>
<th>C.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>irrelevant for the current situation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.997</td>
<td>5.598</td>
<td>35.79</td>
<td>164.57</td>
<td>30.70</td>
<td>5.36</td>
<td>195.27</td>
<td>0.958</td>
</tr>
<tr>
<td>2</td>
<td>6.425</td>
<td>5.140</td>
<td>37.37</td>
<td>154.71</td>
<td>47.23</td>
<td>3.28</td>
<td>201.94</td>
<td>0.972</td>
</tr>
<tr>
<td>3</td>
<td>5.989</td>
<td>4.791</td>
<td>38.90</td>
<td>147.47</td>
<td>64.39</td>
<td>2.29</td>
<td>211.86</td>
<td>0.982</td>
</tr>
<tr>
<td>4</td>
<td>5.641</td>
<td>4.513</td>
<td>40.44</td>
<td>142.04</td>
<td>86.08</td>
<td>1.65</td>
<td>228.12</td>
<td>0.988</td>
</tr>
<tr>
<td>5</td>
<td>5.341</td>
<td>4.273</td>
<td>42.14</td>
<td>138.13</td>
<td>125.08</td>
<td>1.10</td>
<td>263.21</td>
<td>0.988</td>
</tr>
</tbody>
</table>

$^1\Theta_I^*$ same as in Table 2.
TABLE 7: Effects of $\sigma_\alpha$ on the Optimal Solutions;
\(\Theta^*_M\)—Maximizing Volume-Discounting Manufacturer; Iso-elastic \(D(p)\)

<table>
<thead>
<tr>
<th>$\sigma_\alpha$</th>
<th>$w^*$</th>
<th>$w_d^*$</th>
<th>$Q_d^*$</th>
<th>$\Theta^*_M$</th>
<th>$\Theta^*_R$</th>
<th>$\Theta^<em>_C = \Theta^</em>_M + \Theta^*_R$</th>
<th>$\Theta^*_I$</th>
<th>C.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14.84</td>
<td>1.62</td>
<td>61.28</td>
<td>38.02</td>
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<td>43.35</td>
<td>43.60</td>
<td>0.994</td>
</tr>
<tr>
<td>0.1</td>
<td>15.74</td>
<td>1.85</td>
<td>40.87</td>
<td>34.61</td>
<td>6.46</td>
<td>41.07</td>
<td>44.00</td>
<td>0.933</td>
</tr>
<tr>
<td>0.3</td>
<td>18.23</td>
<td>2.01</td>
<td>31.31</td>
<td>31.53</td>
<td>6.89</td>
<td>38.42</td>
<td>44.69</td>
<td>0.860</td>
</tr>
<tr>
<td>0.4</td>
<td>18.30</td>
<td>2.15</td>
<td>24.83</td>
<td>28.51</td>
<td>7.26</td>
<td>35.77</td>
<td>45.66</td>
<td>0.783</td>
</tr>
<tr>
<td>0.5</td>
<td>17.85</td>
<td>2.15</td>
<td>24.63</td>
<td>25.87</td>
<td>7.26</td>
<td>33.13</td>
<td>46.95</td>
<td>0.706</td>
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<tr>
<td>0.6</td>
<td>15.94</td>
<td>2.01</td>
<td>30.24</td>
<td>24.15</td>
<td>8.13</td>
<td>32.28</td>
<td>48.58</td>
<td>0.664</td>
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<tr>
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<td>12.10</td>
<td>1.83</td>
<td>41.47</td>
<td>23.09</td>
<td>9.58</td>
<td>32.67</td>
<td>50.62</td>
<td>0.646</td>
</tr>
<tr>
<td>0.8</td>
<td>10.31</td>
<td>1.68</td>
<td>59.11</td>
<td>22.73</td>
<td>11.47</td>
<td>34.2</td>
<td>53.17</td>
<td>0.643</td>
</tr>
</tbody>
</table>

APPENDIX 1: SOME LEGAL ASPECTS OF THE [pm] ARRANGEMENT THAT ARE OVERLOOKED IN THE QUANTITATIVE SUPPLY CHAIN LITERATURE

Fixing a retail price at $p_M$ by a “supplier” (or “manufacturer”), popularly known as “resale price maintenance,” is often perceived in the form of minimum price maintenance -- an illegal practice in many countries (see, e.g., Deneckere, Marvel & Peck 1997, Flath & Nariu 2000, among numerous others). In contrast, it is not illegal to fix a maximum retail price — which is in effect our [pm] arrangement, since the optimal retail price set by the (non-cooperative) game-playing retailer will always be higher than the retail price needed to maximize the channel profit (the “double marginalization” principle). For the United States, in “State Oil Co. v. Khan,” in which a retailer sued a supplier for imposing a maximum gasoline retail price, an unanimous 1997 Supreme Court explicitly held that suppliers do not violate antitrust laws by setting a maximum allowable retail price; see, e.g., U.S. Federal Trade Commission website [http://www.ftc.gov/ogc/briefs/khan.htm](http://www.ftc.gov/ogc/briefs/khan.htm), or Levy & Weitz (2001, pg. 483 and pg. 714, note #38). In the European Union, Regulation 2790/99 of the European Commission (see, e.g., Gogeshvili 2002) explicitly exempts maximum retail price maintenance from antitrust prohibitions. Similarly, most developed Asian economies (e.g., Japan, Hong Kong) do not prohibit maximum price maintenance.

Nevertheless, although maximum resale price restriction (i.e., [pm]) exists in the real world, as exemplified in “State Oil Co. v. Khan”, it is not very common. Also, although minimum resale price maintenance has been the subject of many theoretical analyses, relatively few theoretical studies of [pm] exist (see, e.g., the references in Reiffen 1999 and...
http://www.ftc.gov/ogc/briefs/khan.htm. Sections 5 to 7 of this paper demonstrate: (i) the merits of \[pm\] as a normative channel-coordination arrangement; and (ii) its ability to rationalize the non-cooperative gaming structure. Therefore, \[pm\] should receive more attention from theoreticians, practitioners and government regulators.

Besides imposing \[pm\] rigidly, for many products (though not for, say, the gasoline involved in “State Oil Co. v. Khan”), the manufacturer can print a “suggested retailer price” or a “list price” on the product. In many case this can effectively pressure the retailer to charge no more than the labeled price.

APPENDIX 2: DERIVING THE $\Theta_M$ (EQN. 15) AND $\Theta_R$ EXPRESSIONS IN AN [mS] QUANTITY-DISCOUNTING SYSTEM; LINEAR DEMAND CURVE

The quantity discount scheme is defined by the 3-tuple $(w, w_d, Q_d)$; i.e., the wholesale price/unit is reduced from the regular level of $w$ to the discounted level of $w_d$ if the order quantity equals or exceeds the “qualifying” quantity $Q_d$. The demand curve is $\hat{\Delta}(p) = D_o - \hat{b}(p-p_o) = \tilde{a} - \tilde{b}p$; while $\hat{b}$ is known to the manufacturer only as a random variable, the retailer acts after the actual $b$-value (and hence the $a$-value) is observed.

For any given values of $(w, w_d, Q_d)$ declared by the manufacturer, the profit-maximizing retailer will respond as follows (after having observed the actual $b$-value):

First compute $Q_T = (a-bw)/2$. $Q_T$ is the profit-maximizing order quantity for the retailer; assuming that no “qualifying” order quantity $Q_d$ has been imposed; the formula is adapted from the $V_1^*$-formula in Table 1. If $Q_T \geq Q_d$, handle it as “Situation 1” (explained below), otherwise handle it as “Situation 2.”

**Situation 1.** If $Q_T \geq Q_d$, the retailer’s order quantity and retail price will be

$$Q_1 = (a-bw)/2; \quad p_1 = (a+bw)/2$$

(adapted from Table-1’s $p_1^*$-formula). (A1)

Hence, the profits of the two players would be:

$$\Theta_{R1} = \frac{Q_1(p_1-w)}{4}\frac{(a-bw)^2}{4b}; \quad \Theta_{M1} = \frac{Q_1(w-c)}{2}. \quad \text{(A2)}$$

**Situation 2.** If $Q_T < Q_d$, the retailer has to consider two alternatives:

**Alternative X:** forgo the discount-price opportunity; the relevant expressions are then:

$$Q_X = (a-bw)/2; \quad p_X = (a+bw)/(2b);$$

$$\Theta_{RX} = \frac{Q_X(p_X-w)}{4b}; \quad \Theta_{MX} = \frac{Q_X(w-c)}{2}. \quad \text{(A4)}$$

**Alternative Y:** order $Q_d$ units to take advantage of the lower wholesale price; then set the retail price $p_Y$ such that the $Q_d$ units can be sold; i.e., set $p_Y$ such that $a-bp_Y = Q_d$. The order-quantity, price and profit expressions are then:

$$Q_Y = Q_d; \quad p_Y = p_0-(Q_d-D_o)/b; \quad \text{ (A5)}$$

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\[ \Theta_{RY} = Q_d(p_Y - w_d); \quad \Theta_{MY} = Q_d(w_d - c). \] (A6)

Obviously, the retailer implements Alternative X if \( \Theta_{RX} \geq \Theta_{RY} \).

Straightforward manipulations with (A3), (A4), (A5) and (A6) show that the conditions for Alternatives X and Y can be stated as follows:

Implement Alternative X if \( \{b > B\} \) or \( \{b < C\} \); \[ \{C < b < B\} \] (A7)

Implement Alternative Y if \( \{b < B\} \); \[ \{C < b < B\} \]

where \( B_1 = 2Q_d(w - w_d)[D_o(w - w_d) + (2p_o - w - w_d)(2Q_d - D_o)] \] (A8)

\[ B = \{D_o(w - w_d) + (2Q_d - D_o)(p_o - w_d) + \sqrt{B_1}\}/(p_o - w)^2 \] (A9)

\[ C = \{D_o(w - w_d) + (2Q_d - D_o)(p_o - w_d) - \sqrt{B_1}\}/(p_o - w)^2 \] (A10)

Furthermore, the condition “\( Q_T \geq Q_d \)” (or “\( [(a - bw_d)/2] \geq Q_d \)” is equivalent to the conditions:

\[ \text{“} b \geq L_1 \text{” if } p_o > w_d; \quad \text{“} b \leq L_1 \text{” if } p_o < w_d; \text{ where } L_1 \equiv (2Q_d - D_o)/(p_o - w_d). \] (A11)

To summarize, the manufacturer’s profit \( \Theta_M \) will be:

(i) When \( p_o > w_d \), \( \Theta_M \) is

\[ \Theta_{M1} \text{ if } \{\infty > b \geq L_1\}; \]
\[ \Theta_{MX} \text{ if } b \in (\{b > B\} \cup \{-\infty < b < C\}) \cap \{b < L_1\} \equiv L_2; \]
\[ \Theta_{MY} \text{ if } b \in \{C < b < B\} \cap \{b < L_1\} \equiv L_3. \] (A12)

(ii) When \( p_o < w_d \), \( \Theta_M \) is

\[ \Theta_{M1} \text{ if } \{-\infty < b \leq L_1\}, \text{ or } \]
\[ \Theta_{MX} \text{ if } b \in (\{b < C\} \cup \{B < b < \infty\}) \cap \{b > L_1\} \equiv L_4; \]
\[ \Theta_{MY} \text{ if } b \in \{C < b < B\} \cap \{b > L_1\} \equiv L_5. \] (A13)

Consider now the situation where \( p_o = w_d \). The condition “\( Q_T \geq Q_d \)” (or “\( [(a - bw_d)/2] \geq Q_d \)” is now equivalent to “\( Q_d \leq D_o/2 \)” or “\( (2Q_d - D_o) \leq 0 \).” Thus, if the condition “\( (2Q_d - D_o) \leq 0 \)” holds, the retailer implants (A1) in “Situation 1.” Otherwise, “Situation 2” applies, and the retailer implements Alternative X if \( \{b > B\} \) or \( \{b < C\} \), but he implements Alternative Y if \( \{C < b < B\} \). The manufacturer’s profit \( \Theta_M \) is therefore:

\[ \Theta_{M1} \text{ if } (2Q_d - D_o) \leq 0; \text{ otherwise } \]
\[ \Theta_{MX} \text{ if } b \in (\{b > B\} \cup \{\infty < b < C\}), \text{ and } \]
\[ \Theta_{MY} \text{ if } b \in \{C < b < B\}. \]

However, it can be seen that the preceding expression is equivalent to (A12) because: (i) \( L_1 = -\infty \) if \( (2Q_d - D_o) \leq 0 \) and \( p_o \to w_d + 0 \); and (ii) \( L_1 = +\infty \) if \( (2Q_d - D_o) \geq 0 \). Hence, the validity region
of (A12) can be rewritten as “When \( p_o \geq w_d \).” Incidentally, it can be similarly shown that the validity region of (A13) can be written as “When \( p_o \leq w_d \).”

Therefore, for any given wholesale-pricing scheme \((w, w_d, Q_d)\), the manufacturer’s expected profit is

\[
\Theta_M = \int_{L_1}^{\infty} \Theta_{M1} f_b(x)dx + \int_{L_2} \Theta_{MX} f_b(x)dx + \int_{L_3} \Theta_{MY} f_b(x)dx \quad \text{if} \quad p_o \geq w_d \quad (A14)
\]

\[
\Theta_M = \int_{-\infty}^{L_1} \Theta_{M1} f_b(x)dx + \int_{L_4} \Theta_{MX} f_b(x)dx + \int_{L_5} \Theta_{MY} f_b(x)dx \quad \text{if} \quad p_o < w_d \quad (A15)
\]

Substituting (A2), (A4) and (A6) into the above formulas, we have

\[
\Theta_M = \frac{1}{2} (a - bw)(w-c) + \int_{L_1}^{\infty} \left[ Q_d (w_d - c) - \frac{D_o + x(p_o - w)}{2} (w - c) \right] f_b(x)dx
\]

\[
+ (w_d - w) \int_{L_1}^{\infty} \frac{D_o + x(p_o - w - w_d + c)}{2} f_b(x)dx \quad \text{for} \quad p_o \geq w_d \quad (A16)
\]

\[
\Theta_M = \frac{1}{2} (a - bw)(w-c) + \int_{L_1}^{\infty} \left[ Q_d (w_d - c) - \frac{D_o + x(p_o - w)}{2} (w - c) \right] f_b(x)dx
\]

\[
+ (w_d - w) \int_{L_1}^{\infty} \frac{D_o + x(p_o - w - w_d + c)}{2} f_b(x)dx \quad \text{for} \quad p_o < w_d \quad (A17)
\]

These are stated in the main paper as (15). The manufacturer’s problem is to find the values of \( w, w_d, \) and \( Q_d \) that will maximize \( \Theta_M \).

Following similar arguments, it can be shown that retailer’s expected profit under a given manufacturer’s optimal pricing scheme \((w^*, w_d^*, Q_d^*)\) is

\[
\Theta_R = \int_{L_1}^{\infty} \Theta_{R1} f_b(x)dx + \int_{L_2} \Theta_{RX} f_b(x)dx + \int_{L_3} \Theta_{RY} f_b(x)dx.
\]

\[
= (a - bw^*)^2/(4b)
\]

\[
+ \frac{D_o}{4} \left( E \left( \frac{1}{b} \right) - \frac{1}{b} \right) + \int_{L_1}^{\infty} \left[ \frac{D_o (w^* - w_d^*)}{2} + \frac{(w^* - w_d^*) (2p_o - w^* - w_d^*)}{4} x \right] f_b(x)dx
\]

\[
+ \int_{L_3} \left[ Q_d (p_o - w^*_d) - \frac{(Q_o^* - D_o)}{4x} - \frac{D_o (p_o - w^*_d)^2}{2} - \frac{(p_o - w^*)^2}{4} x \right] f_b(x)dx
\]

\[
\text{if} \quad p_o \geq w_d^* \quad (A18)
\]

\[
\Theta_R = \int_{-\infty}^{L_1} \Theta_{R1} f_b(x)dx + \int_{L_4} \Theta_{RX} f_b(x)dx + \int_{L_5} \Theta_{RY} f_b(x)dx
\]

\[
= (a - bw^*)^2/(4b) + \frac{D_o}{4} \left( E \left( \frac{1}{b} \right) - \frac{1}{b} \right) + \int_{L_1}^{L_4} \left[ \frac{D_o (w^* - w_d^*)}{2} + \frac{(w^* - w_d^*) (2p_o - w^* - w_d^*)}{4} x \right] f_b(x)dx
\]

23
\[
+ \int_{1/5} \left[ Q_d^* (p_o - w_d^*) - \frac{(2Q_d^* - D_o)^2}{4x} - \frac{D_o (p_o - w_d^*)}{2} - \frac{(p_o - w_d^*)^2}{4x} \right] f_b(x) \, dx
\]

if \( p_o < w_d^* \) \hspace{1cm} (A19)

In (A18) and (A19), note incidentally that \( E \left( \frac{1}{b} \right) \) is not the same as \( \frac{1}{b} \) (which is 1/[\( E(b) \)]); see, e.g., further derivations in LL (2005), equations (35) to (36).

APPENDIX 3: DERIVING THE EXPRESSIONS OF \( \Theta_M \) (EQNS 19) AND \( \Theta_R \) IN A [mS] QUANTITY-DISCOUNTING SYSTEM; ISO-ELASTIC DEMAND CURVE

For any value of \((w, w_d, Q_d)\) declared by the manufacturer, the retailer will, after having observed the actual value of \( \tilde{a} \), respond as follows:

First compute \( Q_t = D_o \frac{\alpha(a-1)}{[(w_d \alpha)]^a} \). \( Q_t \) is the profit-maximizing order quantity for the retailer assuming that no qualifying order quantity \( Q_d \) has been imposed; the formula is adapted from the \( \tilde{P}_w \)-formula of (10). If \( Q_t \geq Q_d \), handle it as “Situation 1,” otherwise handle it as “Situation 2.”

**Situation 1.** If \( Q_t \geq Q_d \), the retailer’s order quantity and retail price will be

\[ Q_1 = D_o \frac{\alpha(a-1)}{[(w_d \alpha)]^a}; \quad p_1 = \frac{\alpha w_d}{(\alpha-1)} \] (adapted from Table-1’s \( p_1^* \)-formula) \hspace{1cm} (A20)

Hence, the profits of the two players would be:

\[ \Theta_{R1} = Q_1 (p_1 - w); \quad \Theta_{M1} = Q_1 (w - c). \] \hspace{1cm} (A21)

**Situation 2.** If \( Q_t < Q_d \), the retailer has to consider two alternatives:

**Alternative X:** forgo the discount-price opportunity; the relevant expressions are then:

\[ Q_X = D_o \frac{\alpha(a-1)}{[(w_d \alpha)]^a}; \quad p_X = \frac{\alpha w_d}{(\alpha-1)}; \] \hspace{1cm} (A22)

\[ \Theta_{RX} = Q_X (p_X - w); \quad \Theta_{MX} = Q_X (w - c). \] \hspace{1cm} (A23)

**Alternative Y:** order \( Q_d \) units to take advantage of the lower wholesale price; then set the retail price \( p_Y \) such that \( Q_d \) can be sold; i.e., set \( p_Y \) such that \( a - b \cdot p_Y = Q_d \). The order-quantity, price and profit expressions are then:

\[ Q_Y = Q_d; \quad p_Y = p_o \frac{D_o/Q_d}{1/a}; \] \hspace{1cm} (A24)

\[ \Theta_{RY} = Q_d (p_Y - w_d); \quad \Theta_{MY} = Q_d (w_d - c). \] \hspace{1cm} (A25)

Obviously, the retailer implements Alternative X if \( \Theta_{RX} \geq \Theta_{RY} \).

How the retailer will react ultimately for any given manufacturer-set \((w, w_d, Q_d)\) is stated below:

First define the following symbols and parameters:

\[ h(x) \equiv \frac{w_d^x [(x-1)p_o]}{[(x-1)D_o]^x}; \quad F(x) \equiv D_o p_o \frac{D_o (x-1)}{(wx)^x}; \quad G(x) \equiv Q_d [p_o (D_o/Q_d)^{1/x} - w_d]; \]
$L_o$ is the root to the equation: $h(x) = D_o/Q_d$;

$\alpha_1$ is the root to the equation: $\log_e\{w_d/[p_o(x-1)]\} - 1/(x-1) = 0$;

$\alpha_2$ and $\alpha_3$ are two roots to the equation $h(x) - D_o/Q_d = 0$ when $D_o/Q_d > h(\alpha_1)$;

$\alpha_4$ is an unique root to the equation $F(x) - G(x) = 0$ when $w_d < p_o \leq w$;

$\alpha_5$ and $\alpha_6$ are two roots to the equation $F(x) - G(x) = 0$ when $p_o > w$.

Then it can be shown that (detailed proofs are available from the authors):

1. If $p_o > w_d$, the retailer will implement Situation 1 for $\alpha \geq L_o$; otherwise, (i) when $p_o > w$, the retailer implements Alternative Y (Situation 2) if $\alpha \in \{(1, L_o) \cap (\alpha_5, \alpha_6)\}$ and Alternative X if $\alpha \in \{(1, L_o) \cap (\alpha_5, \alpha_6)\}$; (ii) when $w_d < p_o \leq w$, the retailer will use Alternative X if $\alpha \in \{(1, L_o) \cap (1, \alpha_4)\}$ and Alternative Y if $\alpha \in \{(1, L_o) \cap (\alpha_4, +\infty)\}$.

2. If $p_o = w_d$, (i) the retailer will implement Alternative X if $D_o/Q_d < e$ ($e$ is the natural constant); (ii) if $D_o/Q_d \geq e$, the retailer will implement Situation 1 for $\alpha \geq L_o$ and Alternative X for $\alpha < L_o$.

3. If $p_o < w_d$, (i) the retailer will implement Alternative X if $D_o/Q_d \leq h(\alpha_1)$; (ii) if $D_o/Q_d > h(\alpha_1)$, the retailer will implement Situation 1 for $\alpha_2 \leq \alpha \leq \alpha_3$ and Alternative X for other $\alpha$ values.

As the Stackelberg leader, the manufacturer knows the retailer’s reaction for any given quantity-discount pricing scheme set by her. Therefore, the manufacturer’s expected profit will be

1. If $p_o > w_d$,
   \begin{align}
   \Theta_M &= \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx + \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx + \int_{\alpha_1}^{\alpha_1} \Theta_{MX} f_a(x)dx \quad \text{if } p_o > w \quad \text{(A26.1)}
   \\
   \Theta_M &= \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx + \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx + \int_{\alpha_1}^{\alpha_1} \Theta_{MX} f_a(x)dx \quad \text{if } w_d < p_o \leq w \quad \text{(A26.2)}
   \\
   \Theta_M &= \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx \quad \text{if } D_o/Q_d < e \quad \text{(A26.3)}
   \\
   \Theta_M &= \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx + \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx \quad \text{if } D_o/Q_d \geq e \quad \text{(A26.4)}
   \\
   \Theta_M &= \int_{\alpha_1}^{\infty} \Theta_{MX} f_a(x)dx \quad \text{if } D_o/Q_d < h(\alpha_1) \quad \text{(A26.5)}
   \\
   \Theta_M &= \int_{\alpha_1}^{\alpha_1} \Theta_{MX} f_a(x)dx + \int_{\alpha_1}^{\alpha_1} \Theta_{MX} f_a(x)dx + \int_{\alpha_1}^{\alpha_1} \Theta_{MX} f_a(x)dx \quad \text{if } D_o/Q_d \geq h(\alpha_1) \quad \text{(A26.6)}
   \end{align}
where \( L_1 = (1, \alpha_5, \alpha_6), L_2 = (1, \alpha_5, \alpha_6, \alpha_4) \cap (\alpha_5, \alpha_6), \quad L_3 = (1, \alpha_4] \cap \alpha_4 \) and \( L_4 = (1, \alpha_4, +\infty) \). Equations (A26) are reproduced as equations (16) in the main paper.

Under the optimal pricing scheme of the manufacturer, the retailer’s expected profit is:

1. If \( p_o > w_d \)
   \[
   \Theta_R = \int_{\alpha_5}^{\alpha_6} \Theta_{R_1} f_a(x) dx + \int_{\alpha_4}^{\alpha_6} \Theta_{R_X} f_a(x) dx + \int_{\alpha_5}^{\alpha_6} \Theta_{R_Y} f_a(x) dx \quad \text{if } p_o > w \quad (A27.1)
   \]

2. If \( w_d < p_o \leq w \)
   \[
   \Theta_R = \int_{\alpha_5}^{\alpha_6} \Theta_{R_1} f_a(x) dx + \int_{\alpha_4}^{\alpha_6} \Theta_{R_X} f_a(x) dx + \int_{\alpha_5}^{\alpha_6} \Theta_{R_Y} f_a(x) dx \quad \text{if } w_d < p_o \leq w \quad (A27.2)
   \]

3. If \( p_o = w_d \)
   \[
   \Theta_R = \int_{\alpha_5}^{\alpha_6} \Theta_{R_1} f_a(x) dx \quad \text{if } D_o/Q_d < e \quad (A27.3)
   \]

4. If \( p_o < w_d \)
   \[
   \Theta_R = \int_{\alpha_5}^{\alpha_6} \Theta_{R_1} f_a(x) dx \quad \text{if } D_o/Q_d < h(\alpha_1) \quad (A27.5)
   \]

   \[
   \Theta_R = \int_{\alpha_5}^{\alpha_6} \Theta_{R_1} f_a(x) dx + \int_{\alpha_4}^{\alpha_6} \Theta_{R_X} f_a(x) dx + \int_{\alpha_5}^{\alpha_6} \Theta_{R_Y} f_a(x) dx \quad \text{if } D_o/Q_d \geq h(\alpha_1) \quad (A27.6)
   \]
REFERENCES


