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LARGE IS BEAUTIFUL: HORIZONTAL MERGERS FOR BETTER EXPLOITATION OF PRODUCTION SHOCKS*

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The profitability of horizontal mergers is investigated in a situation in which firms face a production shock and therefore are uncertain about their future costs. I show that, due to production rationalization, small-scale mergers can be profitable if the uncertainty is large. The efficiency gain in production also implies benign welfare consequences. Under cost uncertainty, a profitable merger always improves social welfare if no more than half of the industry's firms are allowed to merge. Finally, I show that the incentives to merge depend on the information structure. Firms are less likely to merge when they possess more information.

I. INTRODUCTION

THE GOAL OF THIS PAPER IS TO INVESTIGATE HOW COST UNCERTAINTY affects the incentives to merge horizontally. In a deterministic world, the profitability of a merger depends on many factors. Using the deterministic case as a benchmark, I study whether a merger is more profitable when production costs are stochastic.

A merger benefits its participants by reducing competition and costs.[1] Salant *et al.* [1983], however, point out that a merger may not always be profitable, as the reaction from non-merged firms is damaging. They show that, absent cost-savings, a merger is profitable only when more than 80 per cent of the industry's firms participate in the merger. This is rather puzzling as it is at odds with the real-life observation of pervasive small-scale mergers. Later developments in merger studies have aimed at solving this puzzle. Scholars have suggested that the reactions from non-merged firms may be beneficial if the firms compete on price (Deneckere and Davidson [1985]) or they may be limited due to decreasing returns to scale (Perry and Porter [1985]) or product differentiation (Qiu and Zhou [2006]). Others have

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[1] Costs can be reduced because of savings on redundant fixed costs or because production is moved to more-efficient firms. Even if costs are not saved, merged firms always soften the competition between them by coordinating their choices.

suggested that the merged firms may achieve cost-savings (Farrell and Shapiro [1990]) or other exogenous benefits (such as becoming Stackelberg leaders as proposed by Daughety [1990] and Levin [1990]). And others still have suggested that the merger scale should be endogenously determined (Kamien and Zang [1990]; Qiu and Zhou [2007]).

All the papers mentioned above analyze merger incentives in a deterministic environment with perfect information. In real life, however, firms may be uncertain about some future market conditions. This would be the case if they face demand or supply shocks. Suppose, for example, that all firms serve the same market but their products need to be shipped from various production sites. Part of the cost of serving the market is therefore the transportation cost. If a shock changes the location of the future market to an unknown place, transportation costs will be stochastic.[2] When the shock eventually comes, firms will adjust their operations according to the realization of the shock, e.g., by serving the market from their closest outlets. Since such flexibility increases with a firm's operation scale, the scale itself provides a rationale for firms to merge. The more outlets a firm has, the better its ability to exploit heterogeneity among the outlets in the face of idiosyncratic shocks.

To demonstrate this point, I build a model in which, due to a future production shock, firms are uncertain about their production costs. Before the uncertainty is realized, a group of firms decides whether or not to merge. If they merge, they will operate several production outlets, the costs of which are necessarily heterogeneous if the shock is idiosyncratic. By shifting production from high-cost outlets to low-cost ones, the merged firms will be able to produce any given quantity of product at a lower cost than will a single firm. This cost advantage tends to make a merger more profitable than in the corresponding deterministic case. I show that even small-scale mergers can be profitable if the cost uncertainty is large, thus providing an alternative solution to Salant *et al.*'s [1983] puzzle. What makes the model particularly appealing is that, unlike many existing models, it explains why two-firm mergers are far more frequent than three- or four-firm mergers. Because the cost advantage increases with the merger scale but at a decreasing rate, the

[2] There are plenty of other examples in which the costs or demand are uncertain. The price of important inputs such as crude oil may fluctuate tremendously. The management team (or, more broadly, any current production facilities) may or may not be the most suitable to carry out new tasks in the future. Buyers in an auction do not know their private valuations of the auctioned item before knowing what that item is. A movie producer is uncertain whether its next release is going to be a big hit. Firms do not know which raw material will be crucial for their next new product before deciding which product will be heavily marketed. In describing the challenges that the Internet brings to businesses, *The Economist* [11/9/00] explains: 'now life has become much more difficult. Change has not only become more rapid, but also more complex and more ubiquitous. Established companies are no longer quite sure who their competitors are, or where their core skills lie, or whether they ought to abandon the particular business that once served them so well.'

profit of the merger per merging party is generally higher for a two-firm merger than for a three-firm merger. Merging therefore is more likely to be individually rational for a two-firm merger than for a three-firm merger. Finally, given the limited merger scale and the efficiency gain in production, I find that a profitable merger always improves social welfare if no more than half of the industry's firms are allowed to merge.

In the main model, I assume that cost information is private at the time of production. By comparing the results with those under two alternative information structures, I show that the incentives to merge depend on the information structure. In particular, firms are less likely to merge when they possess more information. The reason is that mergers are driven by production rationalization under cost uncertainty. When firms have more information, they are able to rationalize their production even without a merger, thus having less incentive to merge.

In the popular media, increased diversity and flexibility in business has been regarded as one of the benefits, and sometimes the driving force, of some mergers. As explained by *The Economist* [12/2/04] in a discussion about recent mergers in the international mining industry, 'consolidation in mining is not just about "bigger means better," nor even bigger means cheaper, because economies of scale can be difficult to capture beyond a certain point. But in mining, size does bring diversity, of commodity, region and political risk, as well as operational flexibility.' In the European paper and pulp industry, consolidation enabled firms to optimize their production by 'grouping similar machines into coherent units and, if necessary, halting production to reflect market conditions. . . . channel[ling] production into units with the best machines for particular products' [*The Economist*, 9/13/01]. When two American oil companies, Conoco and Phillips, merged, analysts pointed out that 'having a broader portfolio of assets has helped merged companies to cope with price swings better than smaller rivals' [*The Economist*, 11/22/01]. Similar remarks can be found everywhere.[3]

Two papers are most relevant to the present study.[4] Gal-Or [1988] finds that demand uncertainty and asymmetric information may hinder mergers.

[3] Speaking of the benefit of a large company in dealing with uncertainties, Dick Parsons, president of Time Warner, remarked (*The Economist*, [11/19/98]): 'When power is moving between different bits of the value chain, you need to own the whole chain.' One of the advantages that multinationals enjoy is flexibility: 'Multinationals' size and scale can make it possible for them to exert power in an exploitative way. . . . A multinational, however, can move production: if America's worker-safety law is too restrictive, the company can move its factory to Mexico' (*The Economist*, [11/20/97]).

[4] These two papers and the present one all assume that information is revealed *after* the merger. In contrast, information may also be revealed *before* the merger. In that case, the merger incentive will also be affected, but in a different way. For example, Das and Sengupta [2001] find that information asymmetry is always a barrier to mergers: an otherwise profitable merger may break down because the uninformed party is not willing to pay the price that the informed party asks.

While Gal-Or studies demand uncertainty, I investigate cost uncertainty. This leads to very different conclusions on the advantages and disadvantages of mergers. Banal-Estanol [2007] also investigates merger profitability under cost uncertainty. He concludes that uncertainty always enhances merger incentives if the signals are privately observed, while I find that the effect is ambiguous. The difference can be traced to the different cost structures: Banal-Estanol assumes that marginal costs are increasing, while I assume that they are constant. Furthermore, I show that Banal-Estanol's findings are not robust. A more detailed comparison with the two papers is presented in Subsection VIII (ii).

When firms merge, they pool their cost information as well, so the present paper is also related to the information sharing literature (Vives [1990]; Raith [1996]). A merger differs from information sharing in that the participants in a merger can reallocate their production directly so that production rationalization takes the most thorough form.[5] Indeed, I argue that there is no information sharing in my model. The extra incentives to merge come solely from production rationalization.

II. THE MODEL

Consider an industry with n risk-neutral firms that produce a homogeneous good and compete on quantities. The firms' marginal costs are constant, but the exact values of the costs are uncertain due to an idiosyncratic production shock. In particular, each firm's marginal cost, c, is assumed to be independently drawn from an identical distribution with p.d.f. $f(c)$, c.d.f. $F(c)$ and expectation $\bar{c}$. The variance of the distribution, σ^2, represents the degree of the uncertainty. There are no fixed costs in production. Demand is $p = a - X$, where X is the total output by the n firms.

The game is played in two stages. In the first stage, without knowing their costs, an exogenously determined number of firms, $k \in [1, n]$, collectively decide whether or not they want to merge into a single firm. They merge if and only if their total profits are expected to increase after the merger.[6] After the merger decision is made, costs are realized. Each firm learns its own cost but not the costs of its competitors.[7] In the second stage, all firms engage in Cournot competition with *incomplete information* about one another's costs.

[5] In the context of increasing marginal costs, Banal-Estanol [2007] has compared the profitability of mergers with that of information sharing.

[6] Because the identity of participant firms (and consequently the scale of the merger) is exogenously given, the merger is called an exogenous merger. By contrast, in endogenous mergers, individual firms make independent choices and thus the membership and scale of the merger are determined endogenously. See, for example, Kamien and Zang [1990].

[7] Two alternative information structures are also considered, in which a firm's cost is known either by all firms or by no firm.

If the merger occurs, the merged firms manage their production facilities together and maximize their joint profits.

III. IF THE FIRMS DO NOT MERGE

We investigate in this section the equilibrium when the k firms do not merge. It is useful to start the analysis with two benchmarks. If firms commit to quantities before the uncertainty is resolved,[8] they are said to possess *zero information*. In that case, each firm's expected profit is

$$\left(\frac{a-\bar{c}}{n+1}\right)^2.$$

Only the expected value of the cost is relevant. A larger uncertainty, in the sense of an increased variance in the cost distribution with the same expected value, will not change the profits. Therefore, uncertainty has no effect; each firm's profit is the same as when costs are deterministic.

Consider the second benchmark. If all the *ex post* levels of costs are revealed to all firms before quantities are chosen, firms are said to possess *complete information*. In that case, each firm's *ex ante* profit will be

$$\left(\frac{a-\bar{c}}{n+1}\right)^2+\frac{(n^2+n-1)\sigma^2}{(n+1)^2}.$$

Only the first two moments of the cost distribution are relevant, and their impacts are conveniently separable and additive. In particular, the effect of uncertainty is fully captured in the second term in the above expression. We can identify three channels through which uncertainty affects a firm's expected profit. First, when a firm's cost varies, its quantity will change accordingly even if all other firms' quantities remain the same. Since a firm's profit is convex in its own cost, uncertainty increases the firm's profit. Second, because firms know one another's costs, when a firm's cost changes, the change will induce a change in all other firms' quantities even if those firms' costs remain the same. Consider firm i. When its cost is high, other firms will produce more, which hurts i. But this occurs when i is producing a small quantity (precisely because its cost is high), so the damage for firm i is relatively small. Conversely, when i's cost is low, other firms will produce less, giving i a benefit that is relatively large due to i's large production quantity. The net effect is that i is better off. Third, i will respond to the cost variations of its rivals. Since it responds in an optimal way (to produce less

[8] In some industries, the production period is long and it is prohibitively expensive to change output quantities once production has started. For example, in airplane or ship manufacturing, orders are taken well before the actual production is carried out; in agriculture, crop quantities are planned before farmers know how effective pesticides or chemical fertilizers are.

when a rival is strong so the disadvantage is reduced, and to produce more when the rival is weak so the advantage is expanded), i benefits from variations in its rivals' costs.[9] In summary, all three channels work in the same direction: greater uncertainty increases *ex ante* profits.

Now we come back to the model's setting of *incomplete information*. Each firm's expected profit is

$$\left(\frac{a-\bar{c}}{n+1}\right)^2+\frac{\sigma^2}{4}.$$

A firm knows its own cost, so it can still adjust its production accordingly. Its rivals' costs, on the other hand, are concealed from the firm and are relevant only to the extent of their expected values.[10] As a result, the variations in its rivals' costs will not affect the firm's quantity choice or expected profit. Uncertainty has no effect on the interaction between firms. Out of the three channels mentioned above under complete information, therefore, only the first channel continues to work when information is incomplete; the other two are shut off. Since the last two channels generate extra profits, the expected profit under incomplete information is smaller than that under complete information, a conclusion well known in the information sharing literature.

In summary, for given expected costs, firms benefit from cost variations because they can adjust their production accordingly. This benefit depends on the information structure. It is the largest when firms have complete information about one another's cost, and it is zero when firms choose quantities before the uncertainty is resolved. For the model's setting of incomplete information, the benefit comes from a firm's knowledge about and hence the adjustment to its own cost; there is no interaction between firms through cost variations.

IV. UNCERTAINTY AND THE INCENTIVES TO MERGE

We now analyze how uncertainty affects the incentives to merge. Suppose that k firms have merged. Since marginal costs are constant, the merged firms will concentrate their production on the firm with the lowest cost.

[9] Although the second and third channels are both about the interaction between firms, they work through very different mechanisms. The second channel is about how rivals respond to a change in a firm's choices, which in turn is a response to the realization of the uncertain parameter, so the sign of the effect depends on both the nature of competition (Cournot or Bertrand) and the source of uncertainty (demand or costs). By contrast, the third channel is about how the firm responds to a change in its rivals' choices, which are simply some profit-relevant parameters. It always increases profits, just like the first channel.

[10] Because demand is linear and marginal costs are constant, a firm's (contingent) quantity is linear in its own cost. Therefore, from the viewpoint of the firm in question, the expected quantities of its rivals depend only on the expected values of their costs.

Call the merged entity firm z. Then, z's marginal cost, c_z, is the lowest value of k independent draws from $f(c)$.

Assumption 1: The pre-merger cost is distributed uniformly over $[c_0 - t, c_0 + t]$ with $c_0 \geqslant t \geqslant 0$ and $c_0 + t \leqslant a$.

As a result, t parameterizes the degree of uncertainty with $t = 0$ representing deterministic (and identical) costs. The following assumption ensures that every firm produces positive quantities under all cost realizations:

Assumption 2: $\frac{t}{a - c_0} \leq \frac{2}{n+1} \equiv t_0$.

Given that $f(c) = \frac{1}{2t}$, the p.d.f. of c_z will be $g(c_z) = kf(c_z)[1 - F(c_z)]^{k-1} = \frac{k(c_0 + t - c_z)^{k-1}}{(2t)^k}$. Denote the expected value of c_z as $\bar{c}_z$ and its variance as σ_z^2. Then,

$$\bar{c} = c_0,\ \sigma^2 = \frac{t^2}{3};\ \bar{c}_z = c_0 - \frac{k-1}{k+1}t,\ \sigma_z^2 = \frac{4kt^2}{(k+1)^2(k+2)}.$$

These expressions will not be used until the next section. For now, they are relevant only to the extent that the following is true:

$$\bar{c}_z \leqslant \bar{c} \text{ and } \sigma_z^2 \leqslant \sigma^2,$$

with equality if $k = 1$ or $t = 0$. That is, both the expected value and the variance of the merged entity's costs are reduced. Define $\delta \equiv \bar{c} - \bar{c}_z = \frac{k-1}{k+1}t$ as the expected cost advantage of the merged entity. This advantage arises from production rationalization among the merged firms – moving production from high-cost firms to low-cost ones – which is desirable only when costs are uncertain and therefore heterogenous. Notice that δ increases in both t and k, so the advantage is larger the larger the uncertainty or the larger the merger scale. In particular, when costs are deterministic ($t = 0$), the advantage disappears.

In the setting with zero information, the merged firms have an expected total profit of

$$\frac{[(a - \bar{c}) + (n - k + 1)(\bar{c} - \bar{c}_z)]^2}{(n - k + 2)^2}.$$

As explained earlier, only expected costs matter. With k being common knowledge, all firms, both merged and non-merged, can figure out the distribution of c_z and choose their quantities accordingly even though the exact values of the costs are unknown. Compared with the deterministic case, the merged firms now have a higher profit both directly from the cost advantage and indirectly from the favorable responses from non-merged firms. Since the pre-merger profits are not affected by uncertainties, the merger now becomes more profitable.

Proposition 1: When costs are uncertain and firms choose quantities before the uncertainty is resolved, a merger is more profitable the greater the uncertainty. In particular, the merger is more profitable than when costs are deterministic.

A merger under cost uncertainty reduces the merged firms' expected cost and therefore tends to be more profitable than when costs are deterministic. This is called the *synergy effect* of uncertainty.

Under complete information, z's expected profit is[11]

$$\frac{[(a-\bar{c})+(n-k+1)(\bar{c}-\bar{c}_z)]^2}{(n-k+2)^2}+\frac{(n-k)\sigma^2+(n-k+1)^2\sigma_z^2}{(n-k+2)^2}.$$

The first term is the same as in the zero-information case, so the synergy effect is still present, but there now appears a second effect. Recall the three channels through which a firm benefits from cost variations. The first and second channels relate to the variation in the firm's own cost, while the second and third channels relate to the interaction between firms. After the merger, z's costs vary to a lesser degree ($\sigma_z^2 \leqslant \sigma^2$). This means that the magnitude of the first and second channels is decreased, making mergers less attractive than in the zero information case. Moreover, the merged entity z has fewer remaining rivals ($n-k$ instead of $n-1$). This reduces the total effect of interactions between firms, lessening the intensity of channels two and three. This, again, makes mergers less profitable in the complete information case. Both make z's benefit from cost variations smaller than that of a single merging firm,[12] and therefore even smaller than that of all k merging firms combined together.

A merger under cost uncertainty reduces the merged firms' benefits from cost variations and therefore tends to be less profitable than when costs are deterministic. This is called the *variation effect* of uncertainty. Because of the variation effect, a merger under complete information is less profitable than that under quantity commitment.

[11] Notice that the pre-merger profit is a special case of the expression with $k=1$, $\bar{c}_z=\bar{c}$ and $\sigma_z^2=\sigma^2$.

[12] It is easy to show that $\frac{(n-k)+(n-k+1)^2}{(n-k+2)^2} \leqslant \frac{(n^2+n-1)}{(n+1)^2}$. Because $\sigma_z^2 \leq \sigma^2$, we have $\frac{(n-k)\sigma^2+(n-k+1)^2\sigma_z^2}{(n-k+2)^2} \leqslant \frac{(n^2+n-1)\sigma^2}{(n+1)^2}$ with strict inequality when $k>1$ and $\sigma^2>0$.

Under incomplete information, z's expected profit is

$$(1) \qquad \pi(k,\ t) = \frac{[(a-\bar{c}) + (n-k+1)(\bar{c}-\bar{c}_z)]^2}{(n-k+2)^2} + \frac{\sigma_z^2}{4}.$$

Compared with the complete information case, the second and third channels through which firms benefit from cost variations are now shut off. As discussed earlier, the two channels reduce merger profitability because the merged entity has a less variable cost and it interacts with fewer rivals. Consequently, a merger under incomplete information is more profitable than that under complete information.[13]

Compared with the zero-information case, the incomplete information case activates the first channel. This profit-increasing channel becomes less important for the merged firm because its effective costs are less variable than the costs of a single firm. Hence the first channel introduces a negative effect on the incentives to merge. Consequently, a merger under incomplete information is less profitable than that under quantity commitment. The ranking of merger profitability in the three information structures is summarized as follows.

Proposition 2: A merger under quantity commitment is more profitable than that under incomplete information, which in turn is more profitable than that under complete information.

Both before and after the merger, uncertainty affects a firm's profit through expected costs and cost variations, the impacts of which are separable and additive. The impact of cost variations (i.e., the variation effect) depends on the information structure, but the impact of expected costs (i.e., the synergy effect) does not. That is why the net effect of uncertainty can be unambiguously ranked between the three information structures. Essentially, cost uncertainty is conducive to mergers because of production rationalization. But production rationalization has already taken place to a certain degree before the merger if firms have some information about their costs. The synergy effect is therefore weakened, the more so the more information firms have.

V. THE NET EFFECT OF UNCERTAINTY

From now on, we focus on the model's setting of incomplete information. Given (1), the merging firms' profits change by (notice that the pre-merger

[13] Banal-Estanol [2007] also finds that the merger incentive under complete information (what he calls 'public information') is always lower than that under incomplete information (what he calls 'private information').

profit can be regarded as a special case of the post-merger profit with $k = 1$):

$$\begin{aligned}\Delta(k, t) &\equiv \pi(k, t) - k\pi(1, t)\\ &= \left\{\frac{[(a-\bar{c})+(n-k+1)\delta]^2}{(n-k+2)^2} + \frac{\sigma_z^2}{4}\right\} - k\left\{\left(\frac{a-\bar{c}}{n+1}\right)^2 + \frac{\sigma^2}{4}\right\}\\ &= \left\{\frac{[(a-\bar{c})+(n-k+1)\delta]^2}{(n-k+2)^2} - \frac{k(a-\bar{c})^2}{(n+1)^2}\right\} - \left\{\frac{k\sigma^2}{4} - \frac{\sigma_z^2}{4}\right\}.\end{aligned} \tag{2}$$

When costs are deterministic, $\delta = 0$ and $\sigma^2 = \sigma_z^2 = 0$, so the merger profitability is $\Delta(k, 0) = \left\{\frac{(a-\bar{c})^2}{(n-k+2)^2} - \frac{k(a-\bar{c})^2}{(n+1)^2}\right\}$. The extra merger profitability due to uncertainty is therefore

$$\begin{aligned}\Omega &\equiv \Delta(k, t) - \Delta(k, 0)\\ &= \left\{\frac{[(a-\bar{c})+(n-k+1)\delta]^2}{(n-k+2)^2} - \frac{(a-\bar{c})^2}{(n-k+2)^2}\right\} - \left\{\frac{k\sigma^2}{4} - \frac{\sigma_z^2}{4}\right\},\end{aligned}$$

in which the first term represents the synergy effect and the second term represents the variation effect.

As explained earlier, when firms have incomplete information, cost variations work only through the first channel, i.e., a firm benefits directly from the variation of its own costs. There is no indirect effect that comes from the interaction between firms.[14] Therefore, the variation effect of uncertainty, captured in the term $\frac{k\sigma^2}{4} - \frac{\sigma_z^2}{4} > 0$, is accounted for by two features. First, firm z's costs are less variable than that of a single merging firm ($\sigma^2 > \sigma_z^2$). Second, the k merged firms respond only to variations in the lowest cost; the remaining $k - 1$ cost variations no longer create value (k in front of σ^2 but not σ_z^2).

These two features arise precisely because the merged firms concentrate their production at the lowest-cost outlet, which is optimal and is responsible for the cost advantage and therefore the synergy effect. It may therefore seem that the synergy effect should always dominate the variation effect; a merger under uncertain costs should always be more profitable than one under deterministic costs. But this is not true. By concentrating their production at the lowest-cost outlet, the merged firms forego their pre-merger benefits from cost variations (the variation effect) in return for the cost advantage (the synergy effect). The benefit of the cost advantage, however, is reduced by the reaction from non-merged firms. It is unclear *a priori* which effect will dominate.[15]

[14] That is why a non-merged firm's benefit from (its own) cost variation continues to be $\frac{\sigma^2}{4}$ after the merger and is not affected by k or σ_z^2. See the expression at the end of Appendix C.

[15] An otherwise profitable action may become unprofitable because the cost of the action is borne fully by the action-takers while the benefit has to be shared with others. This logic is

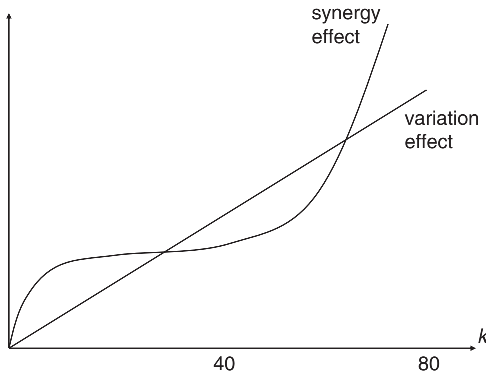

Figure 1
The synergy and variation effects ($n = 80$, $t = 0.02$).

Figure 1 shows how the two effects change with k when $n = 80$ and $t = 0.02$. When k is very small, the variation effect is weak because very few firms merge and therefore lose their pre-merger benefits from cost variations. When k is very large, the synergy effect is strong because there are very few non-merged firms to respond adversely. In both cases, the synergy effect dominates. When k takes an intermediate value, by contrast, the synergy effect is weak (because there are many non-merged firms) while the variation effect is strong (because there are many merged firms), so the variation effect may dominate.

Proposition 3: Cost uncertainty with incomplete information has an ambiguous effect on merger incentives. More specifically, the incentive to merge is enhanced by any uncertainty when $n \leqslant 59$, but, when $n \geqslant 60$, it is hindered by large uncertainty for intermediate values of k.

The variation effect dominates only when both n and t are large. n needs to be large because there need to be many merging firms and many non-merging firms. t needs to be large because the variation effect increases in t at a speed faster than that of the synergy effect.[16] If either n or t is small, then the two curves in Figure 1 will not cross each other and the net effect is always positive. Figure 2 shows the net effect for $n = 80$. When $t < 0.017$, a merger

the same as that in the (un)profitability of mergers under deterministic costs (Salant *et al.* [1983]), except that here we are talking about the *extra* effect on merger profitability that is due to uncertainties.

[16] As a result, for given k, the net effect (and the merger profitability) is non-monotonic in t: it increases in t when t is small and decreases in t when t is large.

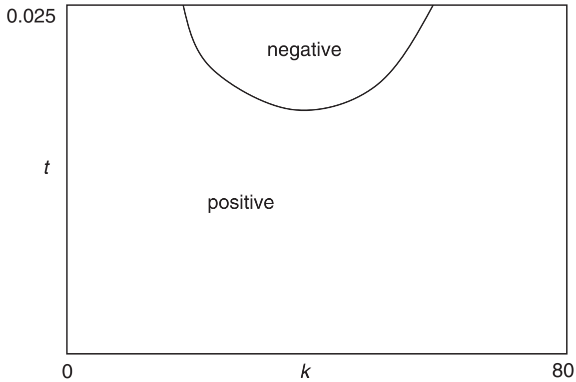

Figure 2
The net effect of uncertainty on the incentives to merge ($n = 80$).

under cost uncertainty is always more profitable than in the corresponding deterministic case. But when $t = 0.02$, a merger under cost uncertainty is less profitable if $33 \leqslant k \leqslant 59$.

VI. THE PROFITABILITY OF MERGERS

We now investigate under what conditions a merger with cost uncertainty is profitable. As the profit functions are homogeneous of degree one in $a - c_0$ and t, $a - c_0$ is normalized to 1 without loss of generality. Given the expected values and variances of the costs derived in the previous section, (2) becomes

$$\Delta(k, t) = \left\{ \frac{\left[1 + (n-k+1)\frac{k-1}{k+1}t\right]^2}{(n-k+2)^2} - \frac{k}{(n+1)^2} \right\} - \left[\frac{1}{12} - \frac{1}{(k+1)^2(k+2)}\right] kt^2. \tag{3}$$

A merger is profitable if and only if $\Delta(k, t)$ in (3) is greater than or equal to zero.

Figure 3 shows Δ as a function of k for different levels of uncertainty ($n = 15$). When there is no uncertainty ($t = 0$), a merger is profitable only if most firms in the industry participate in the merger ($k \geqslant 13$). With some uncertainty, the required participation rate is relaxed ($k \geqslant 12$ for $t = 0.06$). When the uncertainty is large enough, in addition to large-scale mergers, a merger involving only a few firms can also be profitable ($k \leqslant 4$ for $t = 0.12$).

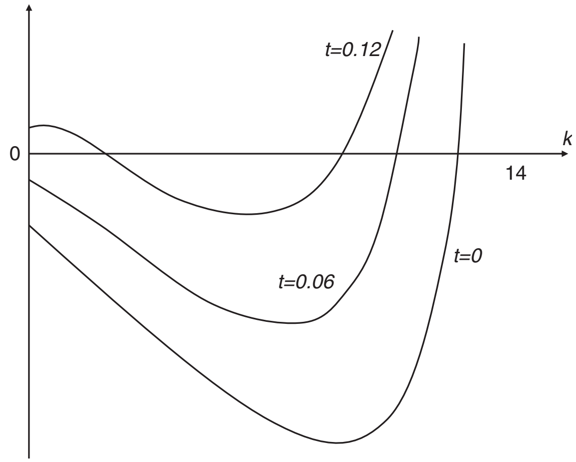

Figure 3
The profitability of mergers at different levels of t ($n = 15$).

Figure 4 shows the areas of profitable and unprofitable mergers in the $k - t$ space for $n = 15$.

The merger scale, k, matters mainly through the interaction between the merged and non-merged firms. In an attempt to reduce the competition between them, the merged firms will behave less aggressively, which induces the non-merged firms to behave more aggressively, hurting the merged firms. When k is very large, few firms are left outside to take advantage of the reduced competition, so the merger is profitable. When k is very small, although the merger is unprofitable due to the reaction of non-merged firms, the loss to the merged firms is small because only a few outlets are shut down. This accounts for the U-shape of the $t = 0$ curve in Figure 3. With cost uncertainty, the profitability is improved for any k (because n is below 60), and a large enough t will be able to elevate the Δ curve above the horizontal axis for small values of k.[17] In other words, a small-scale merger is profitable mainly because of production rationalization under cost uncertainty, while a large-scale merger is profitable mainly because of the greatly reduced competition. With intermediate values of k, the merged firms bear the full cost of shutting down many outlets while the benefit is largely eaten away by non-merged firms, so the merger tends to be unprofitable.

[17] If n is small, t can be large by Assumption 2 ($t \leqslant \frac{2}{n+1}$). It is possible that a very large t can elevate the entire Δ curve above the horizontal axis. In that case, a merger of any scale is profitable.

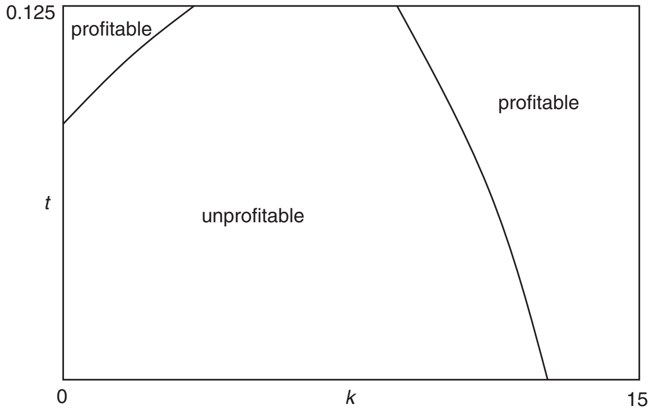

Figure 4
Profitable and unprofitable mergers ($n = 15$).

Salant *et al*. [1983] have studied the profitability of mergers among firms with identical, deterministic costs, i.e., as a special case of $t = 0$ in the present model. Consistent with what is shown in Figures 3 and 4, they find that a merger is profitable only when more than 80 per cent of the industry's firms participate in the merger. In reality, however, we observe mergers only on a much smaller scale; most often, in fact, a merger is between two firms. This paradox has puzzled economists for some time. The above analysis shows that small-scale mergers can be profitable if firms are uncertain about their costs. For example, if $t \geqslant \frac{3\left[(n^2-1)-n\sqrt{n^2-2n+3}\right]}{(2n-1)(n+1)}$, a two-firm merger will be profitable.

In this model, mergers are exogenous in the sense that the merger size, k, is exogenously imposed; firms do not choose k. In real life, however, a merger is most likely the result of negotiations between firms; the firms certainly can control the number of participants. Given that all firms are *ex ante* identical and that the merger decision is made before costs are realized, each participant should expect to have an equal share of the post-merger profits. Therefore, when firms can control the size of the merger, they will choose a k that maximizes $\frac{\pi(k,t)}{k}$. It turns out that $\frac{\pi(2,t)}{2} > \frac{\pi(3,t)}{3}$ for $n > 8$. That is, a two-firm merger is generally more profitable than a three-firm merger. The reason is easy to understand. For an n that is not too small, when $k = 2$ or 3, the profitability of a merger is driven by the merged firms' cost advantage, $\delta = \frac{k-1}{k+1}t$. For fixed t, this advantage is an increasing, concave function in k. When k is larger, more firms will share the benefit of the cost savings, which improve only marginally. As a result, the per-firm profit drops.

The above results are stated in the following proposition:

Proposition 4:

(i) A two-firm merger is profitable if and only if the cost uncertainty is large, i.e., $t \geqslant \frac{3\left[(n^2-1)-n\sqrt{n^2-2n+3}\right]}{(2n-1)(n+1)} \equiv t_2$.

(ii) For $n > 8$, whenever a three-firm merger is profitable, a two-firm merger is also profitable. Furthermore, each participant's profit in the two-firm merger is always larger than that in the three-firm merger.

VII. WELFARE

The expected post-merger social welfare is

$$W(k) = E\left\{\int_0^X (a - X)dX - zc_z - \sum\nolimits_{i=1}^{n-k} x_i c_i\right\},$$

where E is the expectation over all possible costs and $X = z + \sum_{i=1}^{n-k} x_i$ is the total output. The merger changes the expected welfare by $W(k) - W(1)$. We would like to investigate the impact of a privately profitable merger on social welfare.

As shown in the previous section, a merger is profitable only when the merger scale is very large or very small. If large-scale mergers are not allowed (due to, say, antitrust regulations), then a profitable merger will involve only a few firms. This will limit the damage to social welfare. We can show the following:

Proposition 5: If no more than half of the firms are allowed to merge, then

(i) at most four firms will participate in the merger for any n except $10 \leqslant n \leqslant 14$.

(ii) social welfare is always improved by a profitable merger.

A merger reduces competition, but it also makes production more efficient under cost uncertainty. Under the '50% rule,' a profitable merger means the merger scale is very small, so competition is not reduced by much. It turns out that the private and social benefits of mergers are well aligned. In fact, the social benefits are always greater than the private benefits. There are mergers that are socially desirable but privately unprofitable. In that case, the government should subsidize mergers.

Figure 5 shows the privately and socially desirable mergers. The upper-left small corner represents privately profitable mergers, while the whole area to the left of the dashed line represents socially desirable mergers.

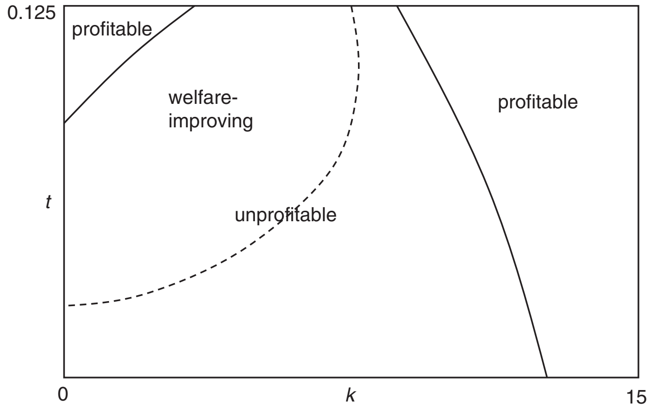

Figure 5
Privately and socially desirable mergers ($n = 15$).

VIII. DISCUSSION AND CONCLUSION

VIII(i). *The meaning of a merger*

What does it mean when firms merge? How do we know that an existing firm is the result of a merger between several single firms or is just a single firm by itself? These questions seem to be at the heart of any merger model. People generally think that a merged firm should be somewhat 'larger' (i.e., more efficient) than a single firm, be it in production, marketing or information collection.[18] Although the idea makes intuitive sense and is frequently alluded to in the popular media, it has yet to be formalized in the economics literature.[19] In this paper, I suggest that when production costs are

[18] 'Intuitively, a merged firm should be 'bigger' than either of the two pre-merger firms . . . ' (McAfee and Williams [1992]). In fact, the merged entity should be bigger than the collection of all merging firms put together. This largeness is usually referred to in the general media as a synergy: 'Synergy exists when the sum of the parts is more productive and valuable than the individual components' (Gaughan [1999], p. 8). Synergies are said to arise from better coordination of complementary businesses (one-stop shopping), cross-marketing of one another's products, spreading overhead, increased specialization of labor and management, more efficient use of capital equipment, expanded product lines, scale economies in production, distribution and advertising, etc. (Gaughan [1999], Chapter 4).

[19] The literature has mainly focused on mergers between identical firms with deterministic costs. When marginal costs are constant, the merged entity is no different from any single merging or non-merging firm (Salant *et al.* [1983]). When marginal costs are increasing (Perry and Porter [1985]) or products are differentiated (Qiu and Zhou [2006]), the merged entity is indeed different from a non-merging firm. The production efficiency, however, is not really improved, as the merged entity is no better at producing any given quantity than is the collection of its members. If each firm incurs a fixed cost, cost-savings can be achieved through a merger, but the merged entity is still identical to any single firm in terms of production. Finally, if marginal costs are constant but different across firms, a merger simply eliminates some less-efficient firms. It does not generate any truly new technology. In the last two cases,

uncertain, a merged entity will be more efficient in production than will any single non-merging firm or the collection of all merging firms, as its total production can be organized among the member outlets in an optimal way – shifted from high-cost outlets to low-cost ones.

A merger enables firms to exploit heterogeneity among participants in the face of production shocks. This logic applies to any demand or supply shock whenever the shock is idiosyncratic. For example, if firms are uncertain about the location of an emerging market, a merged entity is more likely to have a facility that is close to the market than is a firm that owns a single facility. By the same token, a multi-product firm is more likely than a single-product firm to have a market 'hit' that matches consumers' tastes. When facing capacity constraints, a large firm is better able to smooth out production in the face of demand shocks. A large body of buyers (say, a bidding ring in an auction) is more likely to have a high valuation of a given product (McAfee and McMillan [1992]; Waehrer [1999]). Firms may each have a random draw of management talent. A merger enlarges the talent pool, making it more likely for a large firm than for a small firm to find an excellent manager. Similarly, a large population is more likely to generate a capable leader, and a large country is better able to deal with regional natural disasters, such as famine, flood and drought.[20]

Mathematically, the merged entity enjoys a cost advantage because it receives k independent draws of the technology, whereas a non-merged firm gets only one draw.[21] Therefore, another interpretation of the cost uncertainty may be that the technologies are the stochastic realizations of past R&D investments.[22] A merger then means an increase in the number of technological draws. This interpretation of mergers has been utilized by many researchers, especially in the context of auctions (Dalkir *et al.* [2000]; Waehrer and Perry [2003]). There is a caveat for this interpretation,

there are indeed cost-savings even though the merger fails to generate a more-efficient technology. The problem is that the cost-savings are *ad hoc* exogenous and therefore may not be very useful for policies.

[20] 'A large economy is better placed to absorb shocks in different regions. If an oil-price collapse throws Texas into recession, California and New York may still boom. Not only can national taxes provide a regional insurance fund; unemployed Texans can easily move to work in states that have jobs on offer' (*The Economist* [1/1/98]).

[21] The merged entity's cost is the first-order statistic of multiple independent draws. A similar formulation has been used by Waehrer [1999] in modelling the value of bidding rings in an auction and by Dalkir *et al.* [2000] and Waehrer and Perry [2003] in modelling the costs to suppliers in a procurement game. That the merged entity shuts down all but one facility is an artifact due to the assumption of constant marginal costs. If marginal costs are increasing, as in Banal-Estanol [2007], all outlets will be put into use, but production rationalization still takes place.

[22] Oil companies may consider mergers when they are still exploring their oil fields, the productivity of which is unknown at the time of the merger. Pharmaceutical companies may need to make merger decisions without knowing whether their R&D efforts are successful. In general, any investment could result in indeterminate costs in the future, and sometimes a merger occurs before the uncertainty is resolved.

however. The merger decision should be made after the investments are sunk;[23] otherwise, the merger is likely to affect the participants' R&D decisions – they may not continue to invest in all k potential projects.[24]

In financial economics, people think that one benefit of mergers is risk diversification, which is sometimes called coinsurance. The idea is that if earnings are imperfectly correlated, combined earnings will be less volatile (Gaughan [1999]; Penas and Unal [2004]). In that world, volatility is undesirable because firms are risk averse. By contrast, in my model, firms are risk neutral. In fact, volatility (uncertainty) is by all means desirable because it gives firms flexibility in production.[25] As a result, the benefit of a merger is the pooling of production resources and the resulting greater flexibility in the event of demand or supply shocks. The logic for this is totally different from the logic for risk diversification.

VIII(ii). Comparison with the Literature

Comparison with Gal-Or [1988]

Gal-Or [1988] studies whether a merger is more profitable when demands are uncertain and firms receive private signals about demands. By pooling their private signals, the merged firms estimate demands more accurately, making the merger more profitable. To avoid competition, however, the merged firms respond to market signals less aggressively, which induces non-merged firms to be more aggressive. This makes the merger less profitable. Gal-Or shows that the net effect can go either way. Sometimes, 'the merger *may* impose an informational disadvantage on each firm that colludes' (p. 639).

Both Gal-Or and I study whether uncertainty creates any extra merger incentive. We both find that the answer is ambiguous, as uncertainty generates two opposite effects. There are nevertheless important differences. While the uncertainty is on the demand side in Gal-Or's model, it is on the cost side in my model. This leads to different conclusions about the advantages of mergers: a more accurate estimation of demands in her model; a more efficient production technology in my model. Our conclusions about the disadvantages

[23] In the biotech and pharmaceutical industries, drug development is vulnerable to last-minute setbacks due to failures in last-stage clinical trials or rebuffs from the FDA (*The Economist* [6/27/02 and 5/22/03]).

[24] In industries in which R&D plays a crucial role, such as the pharmaceutical industry, analysts generally think that merging firms simply pool their R&D spending; they do not reduce it. For example, the major purpose of the $12.7 billion merger between Bristol-Myers and Squibb in 1989 was to achieve a combined 'annual research and development budget of $600 million,' which led to five new drugs approved by the FDA within two years (Hill [1999] p. 201). According to *The Economist* [1/19/91], the purpose of the merger between computer manufacturers Burroughs and Sperry was to 'pursue the sort of expensive, innovative research neither could afford on its own.'

[25] 'More volatile economies do not appear to invest less as a share of GDP. Countries with greater output variability have enjoyed slightly faster growth in productivity' (*The Economist* [9/26/02]).

of mergers also differ: the informational interaction from non-merged firms in her model;[26] the reduced benefit from cost variations in my model.

Comparison with Banal-Estanol [2007]

Banal-Estanol [2004] studies horizontal mergers under cost uncertainty. He finds that cost uncertainty always enhances the incentives to merge. He further argues that the extra incentive is driven by information sharing, not production rationalization. The different conclusions drawn from my model and Banal-Estanol's come from the cost structure. I assume constant marginal costs, so all merged firms are shut down except for one after the merger. By contrast, Banal-Estanol assumes increasing marginal costs, so all merged firms continue to operate. In reality, it is common for some merged outlets to be shut down.[27] More importantly, merged firms in my model do not share information. Given that a merged firm is shut down, the merged entity will not benefit from the variation of, and therefore the information about, the firm's costs.[28]

In Banal-Estanol's model, the incentives to merge are always enhanced by cost uncertainty because the adverse reaction from non-merged firms is largely limited due to increasing marginal costs. I find that his conclusion is sensitive to underlying assumptions. For example, in the robustness subsection, I show that if the uncertainty is about the slope of the (increasing) marginal costs rather than the intercept, as assumed by Banal-Estanol, the net effect of uncertainty will again be ambiguous.

VIII(iii). *Robustness*

In the main model, I have assumed linear demand, constant marginal costs, uniform cost distribution and an upper bound on the uncertainty. The major conclusions are: (i) Small-scale mergers, which are never profitable when costs are deterministic, will become profitable if the uncertainty is large. (ii) Small-scale mergers improve social welfare whenever they are privately profitable.

[26] For this to happen, the non-merged firms must be able to anticipate the merged firms' actions. They do so by estimating the merged firms' signals. Since a non-merged firm knows only its own signal, the estimation is possible only when signals are correlated. If signals are independent, 'the merger generates no informational effects' (Gal-Or [1988] p. 647). By contrast, I obtain my results when cost signals are totally independent.

[27] Richard Scott, CEO of Columbia Healthcare Corp., explained explicitly when he talked about the merger with Hospital Corp. of America in 1994: 'Some hospitals that are considered inefficient competitors will be acquired simply to eliminate them. We've bought eight hospitals in the last five years solely to shut them down and consolidate them into our operations.' (Hill [1999] p. 292).

[28] The merged firms need to know one another's costs in order to pick up the most efficient technology. But that is production rationalization, not information sharing. Information in my model refers to the deviation of a cost from its expected value, not the cost itself, as expected costs are common knowledge. After the merger, the expected cost of the merged entity, $\bar{c}_z$, becomes known to every firm. The private information that the merged entity has is simply the deviations of its actual costs from its expected cost, $c_z - \bar{c}_z$. In that sense, the merged entity does not possess any more superior information than does a non-merged firm.

(iii) Uncertainty has an ambiguous effect on the incentives to merge; it hinders mergers when the uncertainty is large and there are many merging and non-merging firms. In what follows, I show that these conclusions are mostly robust under alternative assumptions. I do not repeat the verified conclusions here but report only minor discrepancies or inconclusive calculations. The numerical methods and solutions can be found in the Appendix.

Increasing marginal costs
Each firm's total cost is $C_i(x_i) = \frac{c_i}{2}x_i^2$ with c_i being iid from a uniform distribution on $[c_0 - t, c_0 + t]$. When k firms merge, the merged entity's cost will be $C_z(z) = \frac{c_z}{2}z^2$, in which $c_z = 1/\sum_{j=1}^{k}\frac{1}{c_j}$. Conclusions (i) and (iii) hold. For (ii), a privately profitable small-scale merger may hurt welfare if the uncertainty is not very large, but the damage is substantially smaller than when costs are deterministic.

Non-linear demand
The demand is $p = a - X^2$. The first two conclusions hold. The third one cannot be tested due to the difficulty of calculation at high values of k.

Non-uniform cost distribution
The pre-merger cost is binary between $c_0 - t$ and $c_0 + t$ with equal probability. Then, $\bar{c} = c_0$, $\sigma^2 = t^2$, $\bar{c}_z = c_0 - t\left(1 - \frac{1}{2^{k-1}}\right)$, $\sigma_z^2 = \left(\frac{1}{2^{k-2}} - \frac{1}{2^{2k-2}}\right)t^2$. All three conclusions hold except that a privately profitable four-firm merger may hurt welfare when the uncertainty is very large.

Possible shutdown under very large uncertainty
In second-stage Cournot competition, a firm may choose to produce zero when its costs turn out to be very high. Our assumption of $t \leqslant t_0$ avoids this possibility. If $t > t_0$, a merger with intermediate values of k may also be profitable if t is sufficiently large, and a profitable merger is invariably welfare-improving as long as less than half of the firms in the industry participate in the merger.

VIII(iv). *Concluding remarks*

In this paper, I demonstrate that small-scale mergers may be profitable if, at the time of the merger, firms are uncertain about their costs. The paper provides a simple, natural and endogenous formulation of the particular benefits of mergers. Salant *et al.*'s [1983] puzzle implies that mergers purely for market power are mostly unprofitable; a profitable merger must be able to generate particular benefits for the merged firms in one form or another. In this paper, I suggest that a merger pools the participants' production facilities so that they can reallocate the total production among members according to their realized costs. This production rationalization generates a concrete cost advantage. The formulation may then serve as the basis for understanding other merger-related issues, such as endogenous mergers, preemptive mergers, the interaction between horizontal and vertical mergers, etc.

The model generates a few empirically testable hypotheses. For example, a merger is more likely to be formed between firms that face idiosyncratic shocks than between firms that do not. Three-firm mergers are more likely in industries that are already concentrated (i.e., with a small n), while two-firm mergers are more likely in industries that are more competitive (having a large n). The testing of these hypotheses is left for future work.

APPENDIX

A. *Equilibrium under Quantity Commitment*

In all three information structures, the pre-merger equilibrium is a special case of the post-merger equilibrium with $k = 1$ (and consequently $\bar{c}_z = \bar{c}$ and $\sigma_z^2 = \sigma^2$), so we will derive only the post-merger equilibrium.

Firm z's expected profit is $z[a - (n-k)\bar{x} - z - \bar{c}_z]$. The first-order condition leads to $a - (n-k)\bar{x} - 2z - \bar{c}_z = 0$. Similarly, a non-merged firm's first-order condition is $a - (n-k-1)\bar{x} - \bar{z} - 2\bar{x} - \bar{c} = 0$. They yield:

$$\bar{z} = \frac{(a-\bar{c}) + (n-k+1)(\bar{c}-\bar{c}_z)}{n-k+2} \text{ and } \bar{x} = \frac{(a-\bar{c}) - (\bar{c}-\bar{c}_z)}{n-k+2}. \tag{4}$$

The equilibrium expected profit of the merged entity turns out to be $\bar{z}^2$.

B. *Equilibrium under Complete Information*

Let $i = \{1, 2, \ldots, (n-k), z\}$ be the index of all firms after the merger. For firm j, $\pi_j = x_j(p - c_j) = x_j(a - \sum_i x_i - c_j)$. The first-order condition is

$$a - X - x_j - c_j = 0. \tag{5}$$

We sum up the equation over all firms to obtain $X = \frac{(n-k+1)a - \sum_i c_i}{n-k+2}$. Then, (5) implies both $x_j = a - X - c_j$ and $x_j = \frac{a + \sum_i c_i}{n-k+2} - c_j$. The former leads to $\pi_j = x_j^2$, while the latter leads to (after some manipulation)

$$x_z = \frac{[(a-\bar{c}) + (n-k+1)(\bar{c}-\bar{c}_z)] + \sum_{i=1}^{n-k}(c_i - \bar{c}) - (n-k+1)(c_z - \bar{c}_z)}{n-k+2},$$

which means that

$$\begin{aligned} E(\pi_z) &= E(x_z^2) \\ &= \frac{[(a-\bar{c}) + (n-k+1)(\bar{c}-\bar{c}_z)]^2 + (n-k)\sigma^2 + (n-k+1)^2\sigma_z^2}{(n-k+2)^2}. \end{aligned}$$

C. *Equilibrium under Incomplete Information*

Firm z's expected profit is $z[a - (n-k)\bar{x} - z - c_z]$. The first-order condition is

$$a - (n-k)\bar{x} - 2z - c_z = 0. \tag{6}$$

By taking the expectations over z and c_z, the above equation becomes

(7) $$a - (n-k)\bar{x} - 2\bar{z} - \bar{c}_z = 0.$$

Similarly, the first-order condition of a non-merged firm, firm j, is:

(8) $$a - (n-k-1)\bar{x} - \bar{z} - 2x_j - c_j = 0.$$

By taking expectations over x_j and c_j, the above equation becomes

(9) $$a - (n-k+1)\bar{x} - \bar{z} - \bar{c} = 0.$$

Equations (7) and (9) are combined to yield $\bar{z}$and $\bar{x}$, which are identical to those in (4). As a result, from (6) and (8), we get the equilibrium quantities:

$$z(c_z) = \bar{z} - \frac{1}{2}(c_z - \bar{c}_z), \text{ and } x(c_i) = \bar{x} - \frac{1}{2}(c_i - \bar{c}) \text{ for } i = 1, 2, \cdots, n-k.$$

Notice that under Assumptions 1 and 2, $z(c_z) \geqslant 0$ and $x(c_i) \geqslant 0$ for any k and any c_i. The merged entity's profit is

$$\begin{aligned}\pi_z =& z(p - c_z)\\ =& \left[\bar{z} - \frac{1}{2}(c_z - \bar{c}_z)\right]\left\{a - \bar{z} + \frac{1}{2}(c_z - \bar{c}_z) - (n-k)\bar{x} + \frac{1}{2}\sum_{i=1}^{n-k}(c_i - \bar{c}) - c_z\right\}\\ =& \left[\bar{z} - \frac{1}{2}(c_z - \bar{c}_z)\right]\left\{a - \bar{z} - (n-k)\bar{x} - \bar{c}_z - \frac{1}{2}(c_z - \bar{c}_z) + \frac{1}{2}\sum_{i=1}^{n-k}(c_i - \bar{c})\right\}.\end{aligned}$$

Therefore,

$$\begin{aligned}E(\pi_z) =& \bar{z}[a - \bar{z} - (n-k)\bar{x} - \bar{c}_z] + \frac{1}{4}E(c_z - \bar{c}_z)^2\\ =& \bar{z}^2 + \frac{\sigma_z^2}{4}.\end{aligned}$$

Similarly, the expected profit of a non-merged firm is $E(\pi_x) = \bar{x}^2 + \frac{\sigma^2}{4}$.

D. *Proposition 3*

As explained in Section VI, $a - c_0$ can be normalized to 1 without loss of generality. Then, $\Omega = \frac{\left[1+\frac{k-1}{k+1}(n-k+1)t\right]^2 - 1}{(n-k+2)^2} - \left[\frac{1}{12} - \frac{1}{(k+1)^2(k+2)}\right]kt^2$. Obviously, Ω is a quadratic function of t. The sign of Ω is the same as that of $Q = Q(t) = q_2t^2 + q_1t$, where $q_2 = (-k^3 + 7k^2 + 2k - 24)n^2 + (2k^4 - 18k^3 + 32k - 48)n + (-k^5 + 11k^4 - 6k^3 - 16k^2 + 20k - 24)$, and $q_1 = 24(k+1)(k+2)(n-k+1) > 0$. When $n \geqslant 60$ and $k \geqslant 20$, it can be shown that, for any given n, q_2 is always negative while $Q(t_0) < 0$ for intermediate values of k. Given that $t \leqslant t_0$, we conclude that $Q < 0$ (and thus $\Omega < 0$) when t is large while k takes an intermediate value in $[2, n]$. On the other hand, when $n \leqslant 59$, $Q(t_0) > 0$, so Q is positive (and thus $\Omega > 0$) regardless of the sign of q_2.

E. *Proposition 4*

(i) Set $k = 2$ in (3). Δ has the same sign as $\Delta' = (n+1)^2(3-t)[(2n-1)t + 3)] - 18n^2$. The root of Δ' leads to t_2. Note that when $t = t_0$, $\Delta' = 3n^2 + 10n + 1 > 0$, which guarantees $t_2 < t_0$, so Assumption 2 is satisfied.

(ii) Each merged firm's profit is $v(k) = \frac{\pi(k,t)}{k}$. It can be shown that the sign of $v(2) - v(3)$ is the same as that of $f(t) = gt^2 + ht + b$, where $g = -9n^4 + 58n^3 + 11n^2 - 160n + 40 < 0$, $h = -240(n^2 - 3n + 1) < 0$, and $b = 120(n^2 - 6n + 3) > 0$. Therefore, $f(t)$ decreases in t for $t \geqslant 0$. Because $t \leqslant t_0$, we have $f(t) \geqslant f(t_0) = \frac{4(21n^4 - 182n^3 + 11n^2 + 80n + 10)}{(n+1)^2}$, which is positive when $n \geqslant 9$.

F. *Proposition 5*

(i) Consider a profitable merger with $k \geqslant 5$. Then, $n \geqslant 10$ by the requirement $n \geqslant 2k$. The highest k for a profitable small-scale merger is achieved at $t = t_0$. Let $\Delta = \Delta(k, t_0)$. Δ is quadratic in n with $\frac{d\Delta^2}{d^2n} = -8k^3 + 14k^2 + 46k - 12 < 0$, so $\frac{d\Delta}{dn}$decreases in n. We find that $\frac{d\Delta}{dn}\big|_{n=2k} = -2(k+2)(4k^3 - 16k^2 - 11k + 3) < 0$ for $k \geqslant 5$, so that $\frac{d\Delta}{dn} < 0$ for any $n \geqslant 2k$. We also find that $\Delta_{n=2k} = -(k+2)(4k^4 - 20k^3 - 40k^2 - 7k + 3) < 0$ for $k \geqslant 7$, so that $\Delta < 0$ for any $n \geqslant 2k$ and $k \geqslant 7$. Finally, $\Delta_{k=5} = 24(-9n^2 + 152n - 319) < 0$ for $n \geqslant 15$, while $\Delta_{k=6} = 120(-4n^2 + 66n - 175) < 0$ for $n \geqslant 14$. Therefore, a profitable merger can have $k \geqslant 5$ only when $10 \leqslant n \leqslant 14$.

(ii) For a given cost realization, the welfare is $aX - \frac{1}{2}X^2 - zc_z - \sum_{i=1}^{n-k} x_i c_i$. Let $\bar{X} = \bar{z} + (n-k)\bar{x}$. Then, $X = \bar{X} - \frac{1}{2}(c_z - \bar{c}_z) - \frac{1}{2}\sum_{i=1}^{n-k}(c_i - \bar{c})$ so $E(X^2) = \bar{X}^2 + \frac{\sigma_z^2}{4} + \frac{n-k}{4}\sigma^2$. Also, $zc_z = [\bar{z} - \frac{1}{2}(c_z - \bar{c}_z)][\bar{c}_z + (c_z - \bar{c}_z)]$, so $E(zc_z) = \bar{z}\bar{c}_z - \frac{\sigma_z^2}{2}$. Similarly, $E(x_i c_i) = \bar{x}\bar{c} - \frac{\sigma^2}{2}$. Then, the expected welfare is

$$\begin{aligned} W(k) =& a\bar{X} - \tfrac{1}{2}E(X^2) - E(zc_z) - (n-k)E(x_i c_i) \\ =& \frac{(n-k+1)(1+\delta^2) + 2\delta}{n-k+2} - \frac{1}{2}\left(\frac{(n-k+1)+\delta}{n-k+2}\right)^2 + \frac{3}{8}\sigma_z^2 + \frac{3(n-k)}{8}\sigma^2. \end{aligned}$$

The expected welfare before the merger is $W(1) = \frac{(n^2+2n)}{2(n+1)^2} + \frac{3n}{8}\sigma^2$.We can show that $\Delta^W(t) = W(k) - W(1)$ is a quadratic function that increases in t for $t \in [0, t_0]$.

By (i), at most four firms will participate in the merger for any n except for $11 \leqslant n \leqslant 14$. By Proposition 4(i), a profitable merger for $k = 2$ (which implies that $n \geqslant 4$) implies $t \geqslant t_2$. But $\Delta^W(t_2) = \frac{1}{2(2n-1)^2}\left[2(n-2)\sqrt{n^2 - 2n + 3} - 2n^2 + 6n - 3 - \frac{3+12n}{(n+1)^2}\right] > 0$ for $n \geqslant 4$, so $\Delta^w(t) > 0$ when $t \geqslant t_2$. That is, whenever a two-firm merger is profitable, the social welfare will be improved by the merger.

Similarly, if $k = 3$ (which implies that $n \geqslant 6$), we can show that the merger is profitable if and only if

$$t \geqslant \frac{4(n-1)\sqrt{10(13n^2 - 52n + 103)} - 40(n^2 - n - 2)}{(3n^2 - 46n + 63)(n+1)} \equiv t_3.$$

But

$$\begin{aligned} \Delta^W(t_3) =& \frac{(28n^3 - 44n^2 - 108n - 36)\sqrt{10(13n^2 - 52n + 103)}}{(3n^2 - 46n + 63)^2(n+1)^2} \\ &- \frac{-313n^4 + 940n^3 + 1546n^2 - 780n + 1287}{(3n^2 - 46n + 63)^2(n+1)^2} > 0 \end{aligned}$$

for $n \geqslant 6$, so $\Delta^W(t) > 0$ when $t \geqslant t_3$. The case of $k = 4$ or higher (which happens when $11 \leqslant n \leqslant 14$) can be proved in a similar way.

G. *Robustness*

G(1). Increasing marginal costs It can be shown that, in equilibrium, the merged entity produces $z(c_z) = \frac{1}{2+c_z}\frac{a(1-h_x)}{\beta}$and a non-merged firm produces $x_i(c_i) = \frac{1}{2+c_i}\frac{a(1-h_z)}{\beta}$, in which $\beta = h_x(1-h_z)(n-k) + (1-h_x)$ and h_x and h_z are expectations of $\frac{1}{2+c_i}$ and $\frac{1}{2+c_z}$. Normalize $a = 1$. The expected profit of the merged entity is $\pi(k) = \frac{h_z}{2}\left(\frac{1-h_x}{\beta}\right)^2$, while the expected social welfare is $W(k) = \bar{z} + (n-k)\bar{x} - (n-k)\bar{z}\bar{x}$ $-\frac{(n-k)(n-k-1)}{2}\bar{x}^2 - \frac{g_z}{2h_z^2}\bar{z}^2 - \frac{(n-k)g_x}{2h_x^2}\bar{x}^2$, in which $\bar{z}$ and $\bar{x}$ are the expectations of $z(c_z)$ and $x_i(c_i)$. For $n = 15$ and $c_0 = 10$, under deterministic costs ($t = 0$), a merger is profitable if and only if $k \geqslant 6$. With uncertainty, a two-, three-, four-firm merger is profitable if $\frac{t}{c_0} \geqslant 0.85, 0.86$ and 0.83, respectively. A two-firm merger improves welfare if $\frac{t}{c_0} > 9.9$. The damage to welfare by a profitable two-firm merger is 60% of that under the deterministic case. For $n = 40$ and $\frac{t}{c_0} = 1$, the incentives to merge are hindered by uncertainty when $17 \leqslant k \leqslant 30$.

G(2). Non-linear demand The first-order conditions of the merged entity and a non-merged firm, i, are

$$(10) \qquad -3z^2 - 4(n-k)\bar{x}z - c_z + \beta_z = 0,$$

$$(11) \qquad -3x_i^2 - 4[\bar{z} + (n-k-1)\bar{x}]x_i - c_i + \beta_x = 0,$$

in which $\beta_z = a - (n-k)E(x^2) - (n-k)(n-k-1)\bar{x}^2$, $\beta_x = a - E(z^2) - (n-k-1)$ $E(x^2) - 2(n-k-1)\bar{z}\bar{x} - (n-k-1)(n-k-2)\bar{x}^2$, $\bar{z}$and $\bar{x}$ are the expectations of $z(c_z)$ and $x_i(c_i)$, and $E(z^2)$ and $E(x^2)$ are the expectations of $[z(c_z)]^2$ and $[x_i(c_i)]^2$. Take expectations on (10) and (11) to get:

$$(12) \qquad -3E(z^2) - 4(n-k)\bar{x}\bar{z} - \bar{c}_z + \beta_z = 0,$$

$$(13) \qquad -3E(x^2) - 4[\bar{z} + (n-k-1)\bar{x}]\bar{x} - \bar{c} + \beta_x = 0.$$

From (12) and (13) we can express $E(z^2)$ and $E(x^2)$ as functions of $\bar{z}$and $\bar{x}$. Plug these functions into (10) to solve $z = h_z(c_z, \bar{z}, \bar{x})$ and into (11) to solve $x_i = h_x(c_i, \bar{z}, \bar{x})$. Take expectations on both sides of these two expressions to get $\bar{z} = \int_{c_0-t}^{c_0+t} h_z(c_z, \bar{z}, \bar{x})g(c_z)dc_z$ and $\bar{x} = \int_{c_0-t}^{c_0+t} h_x(c_i, \bar{z}, \bar{x})f(c_i)dc_i$. These two equations will solve for the equilibrium $\bar{z}$ and $\bar{x}$. Finally, the equilibria $\bar{z}$ and $\bar{x}$ are plugged into functions h_z and h_x to obtain the equilibrium quantities of the competing firms (as functions of their realized costs). The expected profit of the merged entity will be $\pi = \int_{c_0-t}^{c_0+t} z[a - E(X^2) - c_z]g(c_z)dc_z$, in which $E(X^2) = z^2 + (n-k)E(x^2) + 2(n-k)\bar{x}z + (n-k)(n-k-1)\bar{x}^2$.

For $n = 15$, we place the constraint of $t \leqslant t_0 = 0.23$ to ensure non-negativity of the equilibrium quantities. When costs are deterministic, a merger is profitable only if $k \geqslant 13$. When costs are uncertain, a two-, three-, four-firm merger is profitable if $t > 0.15$, 0.18, and 0.21, respectively, and is welfare improving if $t > 0.02$, 0.03 and 0.04, respectively. A five-firm merger is never profitable for any $t \leqslant t_0$. To verify (iii), we need to calculate the equilibrium at high values of k ($k > 30$). Given that $k-1$ is the exponent of the density function of c_z, the integration that involves the density function is too complicated to carry out, so (iii) cannot be tested.

G(3). Non-uniform cost distribution For any n and $t \leqslant t_0$, a profitable two- or three-firm merger always improves welfare. A profitable four-firm merger may hurt welfare when the uncertainty is large. For example, when $n = 15$ (therefore $t_0 = 0.125$), a two- or three-firm merger is profitable if t is greater than 0.06 and 0.075, respectively, and is welfare-improving if t is greater than 0.008 and 0.011, respectively. A four-firm merger is profitable if $t \geqslant 0.1$, and a profitable four-firm merger improves welfare when $t \leqslant 0.086$ but hurts welfare when $t > 0.086$. If $n = 30$ and $t = t_0$, the incentives to merge are hindered by uncertainty for $9 \leqslant k \leqslant 28$.

G(4). Very large uncertainty Firm z's first-order condition is:

$$z(c_z) = \begin{cases} \frac{1}{2}[a - (n-k)\bar{x} - c_z] & \text{if } c_z \leqslant h_z, \\ 0 & \text{if } c_z > h_z, \end{cases}$$

in which $h_z = \min\{a - (n-k)\bar{x},\ c_0 + t\}$. For given h_z, let $P_z \equiv \Pr(c_z \leqslant h_z) = \int_{c_0-t}^{h_z} g(c)dc$, $\mathring{c}_z \equiv c_z|_{c_z \leqslant h_z}$, $\tilde{c}_z \equiv E(\mathring{c}_z) = \frac{1}{P_z}\int_{c_0-t}^{h_z} cg(c)dc$, and $\tilde{\sigma}_z^2 \equiv E(\mathring{c}_z - \tilde{c}_z)^2 = \frac{1}{P_z}\int_{c_0-t}^{h_z}(c - \tilde{c}_z)^2 g(c)dc$. Also, let $\mathring{z} \equiv z(\mathring{c}_z)$. Then,

$$\text{(14)} \qquad \begin{aligned} \bar{z} &= \frac{P_z}{2}[a - (n-k)\bar{x} - \tilde{c}_z], \\ \mathring{z} &= \frac{\bar{z}}{P_z} - \frac{1}{2}(\mathring{c}_z - \tilde{c}_z). \end{aligned}$$

Similar expressions can be written for a non-merged firm.

There are three possible cases: (I) $c_0 + t \leq a - (n-k-1)\bar{x} - \bar{z} \leq a - (n-k)\bar{x}$, which means that $h_z = h_x = c_0 + t$ and therefore $P_z = P_x = 1$; (II) $a - (n-k-1)\bar{x} - \bar{z} < c_0 + t \leqslant a - (n-k)\bar{x}$, which means that $h_z = c_0 + t$, $h_x = a - (n-k-1)\bar{x} - \bar{z}$, $P_z = 1$ but $P_x < 1$; (III) $a - (n-k-1)\bar{x} - \bar{z} \leq a - (n-k)\bar{x} < c_0 + t$, which means that $h_z = a - (n-k)\bar{x}$, $h_x = a - (n-k-1)\bar{x} - \bar{z}$, $P_z < 1$ and $P_x < 1$. In each case, express h_z and h_x properly and then solve the equilibria $\bar{z}$and $\bar{x}$ from equation (14) and a similar equation for a non-merged firm. These solutions are then plugged back into the expressions of h_z and h_x to verify that this is indeed the case that has been assumed. After $\bar{z}$ and $\bar{x}$ have been determined, firm z's expected profit is $E(\pi_z) = \frac{\bar{z}^2}{P_z} + \frac{P_z}{4}\tilde{\sigma}_z^2$. The expected welfare is $W = a\bar{X} - \frac{1}{2}E(X^2) - E(zc_z) - (n-k)E(x_i c_i)$, in which $\bar{X} = \bar{z} + (n-k)\bar{x}$, $E(X^2) = E(\pi_z) + (n-k)[E(\pi_x) + 2\bar{z}\bar{x} + (n-k-1)\bar{x}^2]$, $E(zc_z) = \bar{z}\tilde{c}_z - \frac{P_z}{2}\tilde{\sigma}_z^2$, $E(x_i c_i) = \bar{x}\tilde{c} - \frac{P_x}{2}\tilde{\sigma}^2$.

For $n = 15$, $t_0 = 0.125$. If $t = t_0$, a merger is profitable for $k \leqslant 4$ or $k \geqslant 10$. If $t = 0.2$, a merger is profitable for any $k \in [2, n]$ and welfare-improving for $k \leqslant 9$. If $t = 0.4$, a merger is profitable and welfare-improving for any $k \in [2, n]$, but welfare is maximized at $k = 7$.

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