Supplementary Material

("Merger waves: A model of endogenous mergers" by Qiu and Zhou,

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In our paper, we have assumed that the merger stage ends if a merger proposal is rejected. In this supplement, we show that our major conclusions are still valid when the assumption is relaxed. A rejection will not end the merger stage. Instead, another proposer is generated, who can make a different proposal. We show that in the specific cases of $n = 3$, 4, 5 and 6, the equilibrium merger configurations are the same as those derived in the main model. We argue that this invariance holds for any n.

Consider the following modification of the merger stage game. At the beginning of each round, any firm can volunteer to make a merger proposal (called the first mover, or FM). There will be only one FM: If there is more than one volunteer, the FM will be selected randomly from all volunteers; if there is no volunteer, the FM will be selected randomly from all firms. The FM thus generated will propose a merger by specifying the merger target and the acquisition price.[1] If a proposal is accepted by the target, the merger takes place, i.e., the acquiree leaves the game with the payment and the remaining firms enter the next round. If the proposal is rejected, a second mover (SM) will be generated through the same mechanism of volunteering and random selection among all firms except the FM. The SM will make a merger proposal to any firm including the FM. This process continues until a merger occurs, which will end the present round and lead the game to the next round. If no merger results after all firms have made their proposals, the whole merger stage is over. Compared to the main model, this game is less restrictive in two respects: (1) a single rejection does not end the merger game; and (2) within each single round, firms can choose to move early or late.

The following notations will be used. Let LM denote the last firm that would move if every

[1] The proposer can always make the proposal unfavorable enough to the target that it is sure to be rejected. This is essentially a "pass". Note that in the main model, "pass" is an explicit option alternative to making a proposal; it is not a special type of proposal.

other firm had proposed and been rejected in that round.[2]

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\begin{array}{rl}
\text{FM}=i: & i \text{ is the FM} \\
i \triangleright \text{FM}: & i \text{ volunteers to be the FM} \\
i \not\triangleright \text{FM}: & i \text{ does not volunteer to be the FM} \\
v^{i\triangleright\text{FM}}: & i\text{'s expected payoff when } i \triangleright \text{FM} \\
i \to j: & i \text{ will propose } i+j \text{ (and request full surplus from it) when LM}=i \\
i \to j \;\; \& \;\; \underline{k \to l}: & i \text{ and } k \text{ both prefer } k \to l \text{ to } i \to j\text{, so } k \to l \text{ will occur} \\
\underline{i \to j} \;\; \& \;\; \underline{k \to l}: & i \text{ and } k \text{ disagree, so both will occur (with equal probability)}
\end{array}
$$

■ **Three Firms**

$N = \{1, 2, 3\}$. When $17 < \alpha \leq 29$, only 1+3 is profitable. Thus, firm 2 will not propose any merger that is acceptable to 1 or 3. In other words, any proposal from firm 2 will be rejected. Call this type of proposal a "passing proposal". Between firms 1 and 3, the one that moves later will propose the merger 1+3 and request the full surplus from the merger, in anticipation that the firm that receives the proposal will have to accept. Therefore, neither firm will volunteer to move until the other has moved. In the unique equilibrium, 1+3 occurs and the two firms expect to split the surplus equally.

When $13 < \alpha \leq 17$, two mergers are profitable: 1+3 and 2+3. Between the two mergers, 1 prefers 2+3 while 2 and 3 both prefer 1+3. If FM=1 and 1's proposal is rejected, 3 will propose to 1 if 3 is the LM, i.e., 3→1. Similarly, 2→3. Because 2 and 3 both prefer 3→1, 2 will volunteer to be the SM while 3 will not, denoted as 2$\triangleright$SM and 3$\not\triangleright$SM. That is, $\underline{3\to1}$ & 2→3. The resulting 3→1 will give zero surplus to 1, so 1$\not\triangleright$ FM. If FM=2, $\underline{3\to1}$ & $\underline{1\to3}$. Both 1 and 3 want to be the LM. The resulting 1+3 will be desirable for 2, so 2$\triangleright$FM. If FM=3, $\underline{2\to3}$ & $\underline{1\to3}$. No matter who is the SM, 3 gets zero surplus, so 3$\not\triangleright$FM. In the unique equilibrium, 2$\triangleright$FM while 1 and 3 do not volunteer (to be the FM or SM). The equilibrium configuration is 1+3.

When $\alpha \leq 13$, all three mergers, 1+2, 1+3 and 2+3, are profitable. For $6.9 < \alpha \leq 13$, each firm's most preferred merger is the one between the other two firms. Furthermore, 1→3, 2→1, and 3→1. If FM=1, $\underline{3\to1}$ & $\underline{2\to1}$, so 1$\not\triangleright$FM. If FM=2, $\underline{3\to1}$ & $\underline{1\to3}$. The resulting 1+3 is the most desirable outcome for 2, so 2$\triangleright$FM. If FM=3, 2→1 & $\underline{1\to3}$, so 3$\not\triangleright$FM. In the unique equilibrium, 2$\triangleright$FM while 1 and 3 do not volunteer. The equilibrium configuration is 1+3.

For $6.6 < \alpha \leq 6.9$, 2's most preferred merger becomes 1+2. If FM=1, 3→1 & $\underline{2\to1}$, so 1$\not\triangleright$FM.

[2] Note that LM may be different from the firm that actually moves the last (i.e., makes a proposal that is accepted).

If FM=2, $\underline{3\to 1}$ & $\underline{1\to 3}$. If FM=3, $\underline{2\to 1}$ & $\underline{1\to 3}$. Now, consider 2 and 3's equilibrium choices about being the FM. If $3\triangleright$FM, $v^{2\triangleright \mathrm{FM}} = \frac{3}{4}\pi_2^{\{1,2\}} + \frac{1}{4}u_2^{N,2+1}$ and $v^{2\ntriangleright \mathrm{FM}} = \frac{1}{2}\pi_2^{\{1,2\}} + \frac{1}{2}u_2^{N,2+1}$. Because 1+2 is 2's most preferred merger, we have $u_2^{N,2+1} > \pi_2^{\{1,2\}}$, so 2's best response is $2\ntriangleright$FM. If $3\ntriangleright$FM, $v^{2\triangleright FM} = \pi_2^{\{1,2\}}$ and $v^{2\ntriangleright \mathrm{FM}} = \frac{1}{2}\pi_2^{\{1,2\}} + \frac{1}{2}u_2^{N,2+1}$. Again, 2's best response is $2\ntriangleright$FM. So $2\ntriangleright$FM is 2's dominant strategy. Given this, $v^{3\triangleright \mathrm{FM}} = \frac{1}{2}\pi_3^{\{1,3\}} + \frac{1}{2}\pi_3^N$ and $v^{3\ntriangleright FM} = \frac{1}{2}\pi_3^{\{1,3\}} + \frac{1}{3}\pi_3^N + \frac{1}{6}u_3^{N,1+3}$. Because $u_3^{N,1+3} > \pi_3^N$, we have $v^{3\ntriangleright FM} > v^{3\triangleright FM}$, so 3's best response is $3\ntriangleright$FM. In the unique equilibrium, none of the three firms will volunteer to be the FM. If FM=1, $3\triangleright$SM; if FM=2 or 3, the remaining two firms will not volunteer to be the SM. The equilibrium configuration is 1+3 and 1+2 with equal probability.

Finally, for $5 \leq \alpha \leq 6.6$, $1\to 2$. If FM=1, $3\to 1$ & $\underline{2\to 1}$, so $1\ntriangleright$FM. If FM=2, $3\to 1$ & $\underline{1\to 2}$, so $2\ntriangleright$FM. If FM=3, $\underline{2\to 1}$ & $\underline{1\to 2}$, so $3\triangleright$FM. The unique equilibrium is that $3\triangleright$FM while 1 and 2 do not volunteer. The equilibrium configuration is 1+2.

In summary, if $\alpha > 29$, no firms merge; if $6.9 < \alpha \leq 29$, 1+3 occurs; if $6.6 < \alpha \leq 6.9$, 1+3 and 1+2 occur with equal probability; finally, if $\alpha \leq 6.6$, 1+2 occurs. The equilibrium is the same as that in the main model except that when $6.6 < \alpha \leq 6.9$ the probabilities of the two mergers (1+3 and 1+2) are slightly different.

■ **Four Firms**

For $n = 4$, there is only one merger configuration, $2+3 \Rightarrow 1+4$, that is profitable when $29 < \alpha \leq 44$. As a result, this configuration will be the unique equilibrium, exactly as in the main model.

■ **Five Firms**

For $n = 5$, we have seen that for $44 < \alpha \leq 59$ there are six profitable merger configurations. Once the first-round merger is finished, the next two rounds will follow a unique path, so we only need to focus on the six possible first-round mergers. They are: (a) 1+2; (b) 1+3; (c) 2+3; (d) 1+4; (e) 2+4; (f) 3+4. Since firm 5 is not involved in these mergers, its strategy in the first round is inconsequential, so let us focus on the choices of firms 1, 2, 3 and 4. It has already been shown that $1\to 4$, $2\to 1$, $3\to 1$, and $4\to 1$. If the LM is 2, 3 or 4, firm 1 will get zero surplus. The only way for 1 to increase its payoff is to increase its chance of being the LM. That means 1 will not volunteer to be the FM, SM or TM (the third mover). Suppose that FM=2. If SM=4, then $\underline{1\to4}$ & $3\to1$, so $4\ntriangleright$SM. If SM=3, then $\underline{1\to4}$ & $\underline{4\to1}$, so $3\rhd$SM. Therefore, if FM=2, $3\rhd$SM. Similarly, if FM=3, $2\rhd$SM. Basically, both 2 and 3 want to move early, so they both volunteer to be the FM. And if they fail to be the FM, they will volunteer to be the SM. Given 1, 2 and 3's strategies, if 4 volunteers to be the FM, SM or TM, 1 will surely be the LM, which will give zero surplus to 4. So 4's strategy is to try to be the LM by not volunteering for any of the first three moves. In the unique equilibrium, 2 and 3 volunteer to be the FM and SM, while 1 and 4 do not volunteer for any move (they try to defer their move as late as possible). The equilibrium merger configuration is $1+4\Rightarrow2+3\Rightarrow1+5$, exactly as in the main model.

■ Six Firms

Now $n=6$. For the first round merger, firm 6 is not involved, while $1\to5$, $2\to5$, $3\to5$, $4\to2$, and $5\to2$. The following observations are useful for determining the equilibrium choices in the first round: (a) 1 prefers 2+4 to $1\to5$; (b) 3 prefers 2+4 to $3\to5$; (c) 4 prefers $4\to2$ to 2+5; (d) 5 prefers $5\to2$ to 2+4; (e) $u_5^{N,5+2} > \frac{1}{2}\left(\pi_5^N + v_5^{N\backslash4}\right)$. By (a), 1 wants to move as early as possible. By (b), 3 wants to move as early as possible. So 1 and 3 will volunteer to be the FM and SM. Now consider 2, 4 and 5's equilibrium choices about being the TM. If TM=2, $4\to2$ and $5\to2$. By (c) and (d), $\underline{4\to2}$ & $\underline{5\to2}$. Then, $2\ntriangleright$TM. If TM=4, $2\to5$ and $5\to2$. By (c), $4\ntriangleright$TM. If TM=5, $4\to2$ and $2\to5$. By (c), $\underline{4\to2}$ & $\underline{2\to5}$. Given that 2 and 4 do not volunteer to be the TM, $v^{5\rhd\text{TM}} = \frac{1}{2}\left(\pi_5^N + v_5^{N\backslash4}\right)$ and $v^{5\ntriangleright\text{TM}} = \frac{1}{3}\left(\pi_5^N + v_5^{N\backslash4} + u_5^{N,5+2}\right)$. By (e), $v^{5\ntriangleright\text{TM}} > v^{5\rhd\text{TM}}$, so 5's best response is $5\ntriangleright$TM. In the equilibrium, 1 and 3 volunteer to be the FM and SM, while 2, 4 and 5 do not volunteer for any move. The first-round merger is 2+4 with probability $\frac{1}{3}$ and 2+5 with probability $\frac{2}{3}$, again exactly as in the main model.

■ General Case

Although our assumption that a single rejection ends the whole merger game sounds strong, the analysis of the above special cases demonstrates that the equilibrium will not change when the assumption is relaxed. For general n, no attempt has yet been made to prove formally the equivalence between the main model and the modified model, but the following intuition may help to argue for such equivalence.

In the main model, a firm's major decision is to pass or to propose. Once a firm makes a proposal, it has all the bargaining power because, by assumption, a rejection will end the game. This will induce some firms to propose when they are drawn. Some other firms, on the other hand, will opt to pass even though they can propose a profitable merger, because they anticipate that an even better outcome will result from some other proposals (of which they are not the targets). In the modified game, the major decision is between whether or not to volunteer for early moves. The last mover will have full bargaining power, because by the time it makes its proposal, every other firm will already have proposed and been rejected. The game will end if the LM's proposal is rejected. As a result, some firms will try to be the LM by not volunteering for early moves. Some other firms, however, will try to move early. Once a firm has moved (i.e., proposing a merger and being rejected), by assumption, it no longer has any chance to propose other mergers. This is like committing to making no further proposals.

Therefore, "propose" in the main model is equivalent to "do not volunteer for early moves" in the modified model, while "pass" in the main model is equivalent to "volunteer for early moves" in the modified model. Furthermore, a firm's optimal choice between the two options does not depend on what happens when a proposal is rejected. The choice is determined by each firm's preference ranking of all profitable mergers.

Given the equivalence, the main model can be viewed as a reduced form of the modified model. The seemingly strong assumption in the main model serves to simplify the calculation, but it would not change the results. It is worth pointing out that both models try to make mergers truly endogenous. Endogenized are not only the merger structure and sequence, but also the order of moves within each round. In the modified model, firms can choose to move early or late. In the main model, firms are drawn randomly, which seems to suggest that the ordering is purely random. But in fact, the order is still endogenous because some but not all firms, depending on each firm's preference ranking of various mergers, will take the "pass" option.