# PROPORTIONAL FEE VS. UNIT FEE: COMPETITION, WELFARE, AND INCENTIVES*

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This paper compares social welfare for a unit versus a proportional fee on competing networks. When demand is sub-convex or isoelastic, proportional fee welfare dominates unit fee and the comparison is independent of network competition. When demand is super-convex, however, unit fee welfare dominates proportional fee if network competition is sufficiently weak. Dominance of unit fee is more likely when network competition weakens or if merchants must single-home. For competing networks, proportional fee is each network's dominant strategy but often leads to a Prisoners' Dilemma that hurts not only networks but also merchants.

## I. INTRODUCTION

Many economic activities involve monetary transfers that can be calculated in two alternative ways: either as a fixed percentage of the transaction value ('proportional fee') or a fixed per-unit amount independent of the transaction value ('unit fee'). A government can levy either *ad-valorem* or specific taxes. In retailing, the agency model uses proportional fee while the wholesale model uses unit fee. Law firms and consulting firms can charge their customers based on commissions or piece rates.

Proportional fee is at the center of several recent, high-profile antitrust cases and its merits *vis-à-vis* unit fee are hotly debated. For instance, merchants have long complained about proportional interchange fee on payment card networks arguing that a per-unit charge is more reasonable because the cost of executing each transaction does not vary much with its price (Shy and Wang [2011]). The ebook industry started with the wholesale model on Amazon's Kindle. Apple then entered the competition by signing agency

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contracts with five major publishers who in turn forced Amazon to adopt the agency model. In 2013, the Department of Justice ordered Apple to abandon the agency agreements.[1] In technology licensing, recent court rulings reveal a trend of supporting per-unit royalties.[2] In 2017, Apple sued Qualcomm for abuse of market power. The suit concerns Qualcomm's practice of collecting patent royalties from device makers rather than component makers, the former proportional fees and the latter unit fees.[3]

Existing studies comparing welfare from proportional and unit fees typically focus on a monopoly network with specific demand functions. Both are restrictive. In real life, network competition is the norm. Visa and MasterCard compete and both face increasing competition from networks such as American Express, PayPal, and UnionPay. Competition between iPad and Kindle drove ebook industry dynamics. In technology licensing, multiple and competing standards persist in many products and industries. Even in public finance, governments increasingly engage in tax competition. In addition, when networks compete, a further complication is the possibility of merchants' multi-homing. For example, ebook publishers sell books on both iPad and Kindle, and most consumers and merchants use both Visa and MasterCard for payments. For demand functions, the literature has focused on sub-convex and isoelastic demands but both theoretical and empirical studies have shown the relevance of super-convex demands, which cannot be ruled out.[4]

This paper investigates the welfare consequence of proportional and unit fees, focusing on the roles of network competition, merchant multi-homing, and demand convexity. For ease of understanding, we shall illustrate the ideas using the payment card industry and refer to card networks and

[1] See United States vs. Apple, Inc., *et al.*, Civil Action No. 1: 12-CV-2826, *https://www.justice.gov/atr/case-document/plaintiff-united-states-final-judgment-and-plaintiff-states-order-entering*.

[2] As cited by Llobet and Padilla [2016], in the 2014 ruling of *Ericsson Inc. v. D-Link Systems, Inc.*, the United States Court of Appeals for the Federal Circuit suggested that *ad valorem* royalties be avoided (*http://www.cafc.uscourts.gov/sites/default/files/opinions-orders/13-1625.Opinion.12-2-2014.1.PDF*). In 2015, DOJ praised the Institute of Electrical and Electronics Engineers (IEEE) for incorporating the per-unit royalty in the organization's updated patent policy (Hesse [2015]).

[3] Qualcomm charges royalties as a percentage of iPhone's retail price, typically at 5%. As pointed out by Llobet and Padilla [2016], component-based royalties are equivalent to unit fee, whereas device-based royalties are equivalent to proportional fee. Viewed from this angle, Apple prefers unit fee while Qualcomm prefers proportional fee. The case was settled by the two parties in 2019, in which Qualcomm maintained its licensing model. See 'Suing for peace,' *The Economist*, Apr. 17, 2019, *https://www.economist.com/business/2019/04/17/apple-and-qualcomm-settle-a-feud-over-patents*.

[4] A sub-convex demand becomes less elastic when the price is lower, or equivalently the pass-through rate is less than 100%, i.e., the percentage increase of price is smaller than the percentage increase of cost (Mrázová and Neary [2017, 2019]). The opposite holds for super-convex demands. For the importance of super-convex demands, see Amir and Lambson [2000], Chen and Riordan [2007], and Zhelobodko *et al.* [2012] for theoretical work, and Ward *et al.* [2002] and Badinger [2007] for empirical studies.

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merchants. However, the results apply directly to other settings such as internet and telecom platforms, retailing, and technology licensing.[5]

We find that when demand is sub-convex or isoelastic, proportional fee welfare dominates unit fee and the comparison is independent of network competition or merchants' homing constraints. When demand is super-convex, however, unit fee dominates proportional fee if network competition is sufficiently weak. In addition, unit fee's dominance is more likely when network competition weakens or merchants are not allowed to multi-home.

The key to understanding these results is that compared to unit fee, proportional fee mitigates double marginalization by reducing a merchant's marginal cost of expanding output. This 'discount effect' is the fundamental force behind all other effects. For a given network fee level, therefore, proportional fee tends to improve welfare. However, anticipating its merchants' incentives to expand, a network wants to raise the fee level which tends to reduce welfare. The tradeoff between these two forces is influenced by all three major factors: demand property, network competition, and merchant homing.

When network competition intensifies, a network's incentive to raise its fee level is weakened as a lower fee allows it to encroach upon rival networks' market shares. Although this business-stealing incentive exists in both fee schemes, it is stronger under proportional fee because merchants' increased earnings from business stealing can be (partially) captured by its network only under proportional fee. When merchants multi-home, a network will reap two additional benefits if it lowers its fee level. First, every merchant will reallocate its transactions across networks toward the one with the lowest fee. Second, the lower fee level will allow the network's merchants to earn more profits on competing networks under proportional fee but not unit fee, which will again be partially captured by the network under proportional fee. Both effects make it more likely for proportional fee welfare to dominate unit fee.

Most existing studies conclude that on a monopoly network proportional fee welfare dominates unit fee if demand is sub-convex (Gaudin and White [2014, 2020]; Llobet and Padilla [2016]; Johnson [2017]) or isoelastic (Shy and

[5] The model is about two-sided markets where both consumers and merchants pay service fees. However, price neutrality suggests that the equilibrium depends on a combination of the two sides' fees rather than their individual values (note that price neutrality is valid only when the two sides pay the same type of the fee, which is indeed the case in our model). This means that, without any loss of generality, we can focus on a one-sided fee (i.e., the other side pays zero fee), which is mathematically equivalent to vertical relationships. Nevertheless, we would like to keep the word 'network' because the analysis does apply to a network (of, say, payment cards) or platform.

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Wang [2011]).[6] This paper focuses on competitive networks with single or multi-homing merchants. A key message arising from our analysis is that the welfare comparison is ambiguous with more general assumptions and the comparison increasingly favors proportional fee as competition intensifies among networks or merchants. The conditions determining relative welfare are directly useful to antitrust authorities. For example, we find that proportional fee tends to be socially undesirable if demand is super-convex, network or merchant competition is weak, merchant cost is large, or merchants cannot multi-home. Therefore, when facing proportional fee complaints, the antitrust authority may either force a switch to unit fee or, even better, encourage network competition or merchant multi-homing.

This paper also compares the performance of networks and merchants under the two fee schemes, which is useful for understanding which fee scheme will arise in equilibrium. When network competition is weak, networks always favor proportional fee but merchants may prefer unit fee. Among the three parties, networks are most enthusiastic about proportional fee, followed by consumers, and then merchants. If network competition is strong, proportional fee reduces the joint profit of networks and merchants. Therefore, networks and merchants may separately prefer proportional fee but will not jointly do so.

Proportional fee has become increasingly popular in the digital marketplace. Existing studies have explained this, in a monopoly environment, by its role in mitigating double marginalization (Shy and Wang [2011]) or ability to price discriminate against merchants (Wang and Wright [2017]). This paper offers an alternative explanation when networks compete. Proportional fee is each network's dominant strategy even though they may be collectively worse off. In fact, such a prisoners' dilemma hurts not only networks but also merchants. This finding explains widespread merchant complaints about proportional fee.

The paper is organized as follows. After setting up the model in Section II, we offer an example in Section III to highlight the key results. The next two sections deal with single-homing merchants. Section IV offers an equilibrium characterization and Section V a welfare comparison. Multi-homing merchants are analyzed in Section VI, followed by Section VII on the profitability

[6] Gaudin and White [2014] focus on the effect of complementary device in the ebook industry. Llobet and Padilla [2016] study patent licensing and innovation. Johnson [2017] mainly investigates retailing business models and the effect of MFN's. Gaudin and White [2020] find that the welfare comparison can be overturned if a monopoly platform charges users for access. In the taxation literature, the general conclusion is that *ad-valorem* tax always raises welfare and is a Pareto improvement if merchant is monopoly (Suits and Musgrave [1953]; Bishop [1968]; Delipalla and Keen [1992]; Skeath and Trandel [1994]; and Anderson *et al*. [2001]). Because a government (equivalent to our network) does not try to maximize its tax revenue, the conclusion is not directly comparable to this study.

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of networks and merchants. Section VIII concludes. All the proofs, detailed equilibrium characterizations, and numerical examples are collected in the Appendix.

## II. THE MODEL

Consumers and merchants must use payment cards to settle their transactions. There are $n \geq 2$ competing networks of the cards and networks' marginal cost of service is normalized to zero.[7] All merchants have the same constant marginal cost of production, $\kappa_M > 0$.

A merchant may operate on a single network ('single-homing') or all networks ('multi-homing'). To make the two settings comparable, we assume the number of merchants transacting on each single network is the same $m$ in both cases. Therefore, the total number of merchants in the whole industry is $n \times m$ in single-homing, but $m$ in multi-homing. The main difference is that a multi-homing merchant's output choice on one network will affect its profits on other networks, whereas a single-homing merchant does not have such concerns.

Following Delipalla and Keen [1992], Skeath and Trandel [1994], and Shy and Wang [2011], we assume merchants carry out Cournot competition in the product market. Consumers differentiate between networks, but not merchants.[8] As such, the inverse demand for goods sold on network $j$ is denoted by $p_j(Q_j, \mathbf{Q}_{-j})$, where $Q_j$ is the total quantity transacted on network $j$, and $\mathbf{Q}_{-j}$ is the vector of the transaction quantities processed by networks other than $j$. For simplicity,[9] we assume:

$$p_j(Q_j, \mathbf{Q}_{-j}) = p(Q_j + \alpha \sum_{-j} Q_{-j}), \;\; j \in \{1, 2, \ldots, n\},$$

where $\alpha \in [0, 1]$ measures the differentiation among network services. When $\alpha = 0$, the setting is equivalent to $n$ independent networks, referred to as monopoly networks.

Because the competing networks are symmetrically differentiated, $Q_j = Q_{-j}$ in equilibrium, which can be completely characterized by a *virtual output*: $[1 + (n-1)\alpha]Q_j = kQ_j$, where

[7] All the qualitative results remain unchanged if network marginal cost is positive. The mathematical analyses presented in the appendix are for the general case where network cost is non-negative.

[8] Products sold on the same network (by different merchants) are regarded as perfect substitutes, while products sold on different networks (by the same or different merchants) are regarded as imperfect substitutes.

[9] The restriction on the demand function is very mild. It implies that network services are symmetrically differentiated and the relative differentiation is a constant (as captured by the parameter $\alpha$).

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$$k \equiv 1+(n-1)\,\alpha$$

captures the degree of network competition, with $k=1$ (i.e., $\alpha=0$) for monopoly networks. When there are more networks ($n$ is larger) or their services become closer substitutes ($\alpha$ is larger), $k$ will increase to indicate intensified network competition.

Suppose that $p(x)$ is twice continuously differentiable with $p(x)>0$ and $p'(x)<0$. Assume that the elasticity of demand $\varepsilon \equiv -p(x)/[p'(x)x] \geq 1$ so that marginal revenue is non-negative, and the concavity of demand $\rho \equiv [p''(x)x]/p'(x) > -2$ so that marginal revenue decreases with output.[10] Of particular interest will be the sign of $\varepsilon\,(\rho+1)+1$, which represents the convexity of the demand function. According to Mrázová and Neary [2017, 2019], a demand is sub-convex if $\varepsilon(\rho+1)+1>0$, isoelastic if $\varepsilon(\rho+1)+1=0$, and super-convex if $\varepsilon(\rho+1)+1<0$.

Each network charges service fees to consumers and merchants who transact on the network.[11] A fee can take two forms: it can be proportional to the transaction price ('proportional fee'), or a per-unit fixed amount ('unit fee'). Denote the unit fee on network $j$ as $t_j$, and the commission rate of a proportional fee scheme as $\tau_j$. For every unit sold at price $p_j$, under unit fee, network $j$ receives $t_j$ while its merchants receive $p_j - t_j$. Under proportional fee, network $j$ receives $\tau_j p_j$ and its merchants obtain $\left(1-\tau_j\right)p_j$.

The game proceeds as follows. For any exogenously given homing setting (single-home or multi-home) and fee scheme (unit fee or proportional fee),[12] all networks simultaneously choose their independent fee levels. Taking all the networks' fees as given, each merchant chooses its output in Cournot competition, and consumers make their purchases. Finally, each network collects service fees based on the transaction price and quantity.

## III. AN EXAMPLE

The analysis can be quite involved for the general demand and it helps to first present our key results in an example. Suppose there are two competing networks ($n=2$) and each network has two merchants ($m=2$). Merchant marginal cost is $\kappa_M=0.1$. The inverse demand for goods sold on network $j\in\{1,2\}$ is

[10] In general, in Cournot competition with $m$ competitors, a non-negative marginal revenue requires $\varepsilon \geq 1/m$, and decreasing marginal revenue requires $\rho > -2m$. We made the more stringent assumptions so that our results would survive even under the most restrictive conditions.

[11] A fee can be charged to either merchants or consumers or both. However, as long as the two sides pay the same type of fees, price neutrality holds (Rochet and Tirole [2002]), and the equilibrium depends only on a combination of the two sides' payments.

[12] In the main analysis, the fee scheme is exogenous and is the same for all networks. Later we will allow networks to choose their fee type independently.

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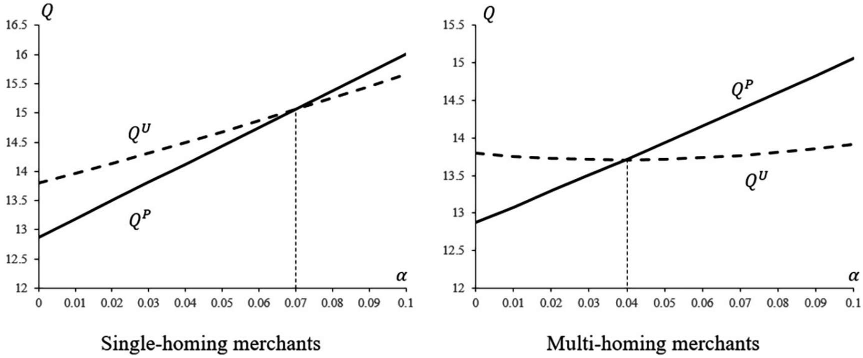


Figure 1
Example: The Impacts of Network Differentiation and Multi-Homing

$$p_j(Q_j, Q_{-j}) = \left[(Q_j + \alpha Q_{-j}) - 1\right]^{-0.5},$$

or $p(x) = (x-1)^{-0.5}$ where $x = Q_j + \alpha Q_{-j}$. Then $\varepsilon = 2(x-1)/x$, $\rho = -3x/[2(x-1)]$, and $\varepsilon(\rho+1)+1 < 0$, i.e., the demand is super-convex.[13]

Because social welfare increases monotonically with the output, the welfare comparison between unit and proportional fees is completely characterized by a comparison between the equilibrium virtual outputs under the two fee schemes. Although an explicit solution of the output is intractable, we can easily show in a graph how the output comparison is influenced by network competition (as captured by the network differentiation parameter, $\alpha$). The left panel of Figure 1 shows the situation for single-homing merchants, and the right panel for multi-homing merchants. Two conclusions can be drawn from the graph.

First, if networks do not compete (i.e., $\alpha = 0$), then $Q^U > Q^P$ regardless of the homing constraints, i.e., unit fee welfare dominates proportional fee. As $\alpha$ increases, proportional fee will become socially more desirable when network competition is sufficiently strong. This is because intensified network competition will raise the output faster under proportional fee than under unit fee.[14]

[13] We have skipped sub-convex and isoelastic demands in the example because proportional fee's social desirability, although extended to network competition and merchant multi-homing, is consistent with the literature. Super-convex is chosen to demonstrate the key finding of this paper, i.e., the possibility and conditions for unit fee to welfare dominate proportional fee.

[14] An interesting observation is that with multi-homing merchants, the equilibrium output under unit fee does not necessarily increase with $\alpha$. As $\alpha$ increases, on one hand, networks will charge lower fee levels, which tends to increase the output; on the other hand, a multi-homing merchant will refrain from competing fiercely, as aggressive competition on one network would hurt its own businesses on other networks.

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Second, allowing merchants to multi-home makes it more likely for proportional fee to welfare dominate unit fee. In the graph, $Q^P > Q^U$ requires $\alpha > 0.07$ for single-homing merchants, but only $\alpha > 0.04$ for multi-homing merchants.

## IV. SINGLE-HOMING MERCHANTS: EQUILIBRIUM CHARACTERIZATION

We now analyze the general case starting with single-homing merchants in this and the next sections. This section will characterize the equilibrium in either fee scheme before highlighting the key difference between the two fee schemes, in what we called the discount effect. The next section will then derive the conditions for welfare comparison.

### IV(i). *Unit Fee*

With unit fee, the profit of merchant $i \in \{1, 2, .., m\}$ is $\pi_{ij}^U = \left(p_j - t_j - \kappa_M\right) q_{ij}$. Its first-order condition (FOC), $\partial \pi_{ij}^U / \partial q_{ij} = 0$, leads to:

$$\frac{1}{m} p_j' Q_j + p_j = \kappa_M + t_j, \tag{1}$$

where $Q_j = \sum_{i=1}^m q_{ij}$ is the total output of all merchants on network $j$. The system of $n$ FOC's, one for each network, characterizes a unique mapping from all networks' unit fees to each network's output: $Q_j(t_j, \mathbf{t}_{-j})$.

From (1), network $j$'s average revenue is:

$$t_j = p_j - \kappa_M + \frac{1}{m} p_j' Q_j, \tag{2}$$

and its profit is $\Pi_j^U\left(t_j\right) = t_j Q_j$. Network $j$ choose $t_j$ to maximize its profit, taking other networks' fees, $\mathbf{t}_{-j}$, as given. Assuming negative semidefinite Hessian matrix, network $j$'s FOC gives rise to:

$$Q_j + \frac{\partial Q_j}{\partial t_j} t_j = 0. \tag{3}$$

This is network $j$'s best response to other networks' fees (as $Q_j$ is a function of all networks' fee levels). There are $n$ such best responses, one for each network, and they jointly characterize the equilibrium fee levels.

It will be convenient to characterize the equilibrium in terms of the virtual output, $Q^U \equiv k Q_j^U$, rather than the equilibrium fee, $t$. In equilibrium, $Q^U$ must satisfy:[15]

[15] Detailed mathematical analyses for the whole paper (including equilibrium characterization, proofs, and mathematical derivations) are collected in the Appendix.

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$$\frac{1}{\partial Q_j/\partial t_j}\frac{Q^U}{k}+t=0. \tag{4}$$

### IV(ii). *Proportional Fee*

Under proportional fee, merchant $i$'s profit is $\pi_{ij}^P = \left[\left(1-\tau_j\right)p_j - \kappa_M\right]q_{ij}$, and the FOC gives:

$$\left(1-\tau_j\right)\left(\frac{1}{m}p_j' Q_j + p_j\right) = \kappa_M. \tag{5}$$

The system of $n$ such FOC's characterizes a unique mapping from all networks' commission rates to each network's output: $Q_j(\tau_j, \tau_{-j})$.

Network $j$'s average revenue under proportional fee is then:

$$\tau_j p_j = p_j - \kappa_M + \left(1-\tau_j\right)\frac{1}{m}p_j' Q_j. \tag{6}$$

It chooses $\tau_j$ to maximize its profit, $\Pi_j^P = \tau_j p_j Q_j$, taking other networks' commission rates, $\tau_{-j}$, as given. Again, assuming negative semidefinite Hessian matrix, $j$'s FOC gives rise to:

$$\left\{p_j + \tau_j p_j'\left[\frac{\partial Q_j}{\partial \tau_j} + (k-1)\frac{\partial Q_{-j}}{\partial \tau_j}\right]\right\}Q_j + \tau_j p_j \frac{\partial Q_j}{\partial \tau_j} = 0. \tag{7}$$

The equilibrium virtual output is $Q^P = kQ_j^P$, where $Q_j^P$ is network $j$'s equilibrium output under proportional fee. It can be shown that $Q^P$ satisfies:

$$\frac{\left\{p + \tau p'\left[\partial Q_j/\partial \tau_j + (k-1)\,\partial Q_{-j}/\partial \tau_j\right]\right\}}{\partial Q_j/\partial \tau_j}\frac{Q^P}{k} + \tau p = 0. \tag{8}$$

### IV(iii). *The Discount Effect*

Given any total output by its merchants, $Q_j$, network $j$'s average revenue under unit fee and proportional fee are given by (2) and (6), respectively. They differ by:

$$\tau_j p_j - t_j = \underbrace{-\tau_j \frac{1}{m}p_j' Q_j}_{\text{Discount Effect }(+)} > 0. \tag{9}$$

That is, for any given $Q_j$ and $Q_{-j}$, a switch from unit fee to proportional fee will raise a network's average revenue (and markup). Under unit fee, if a

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merchant sells more, a one dollar drop in the final sales price is fully shouldered by the merchant itself. Under proportional fee, however, a one-dollar drop of the price will result in only a $1-\tau_j<1$ dollar loss for the merchant. Fixing the output level, therefore, proportional fee reduces a merchant's marginal cost. The saved marginal cost, which equals the commission rate, $\tau_j$, multiplies the merchant's markup, $-p_j'Q_j/m$, can then be taken away by the network without reducing the final output.

Therefore, proportional fee raises a network's average revenue for any given total output. We will refer to this property as the *discount effect*. The effect arises as long as individual merchants have market power, i.e., $-p_j'Q_j/m \neq 0$, which holds regardless of the nature of merchant competition, whether Cournot or Bertrand.

A corollary of the discount effect is that, fixing all networks' outputs at arbitrary levels (not necessary in equilibrium), a network's profit increases when it switches from unit fee to proportional fee. The additional profit is:

$$\Delta\Pi_j(Q_j) \equiv \Pi_j^P(Q_j) - \Pi_j^U(Q_j) = -\tau_j \frac{1}{m} p_j' Q_j^2 > 0. \tag{10}$$

The expression says that for any given $Q_j$, proportional fee increases a network's profit by an amount that equals the commission rate ($\tau_j$) times its merchants' joint net earnings ($-p_j'Q_j^2/m$).

*Lemma 1.* On a given network $j$ and for any given outputs $Q_j$ and $\mathbf{Q}_{-j}$, switching from unit fee to proportional fee will turn a portion of merchant profit into network profit.

## V. SINGLE-HOMING MERCHANTS: WELFARE COMPARISON

Since all merchants have identical cost and their products are homogeneous within a network and symmetric across networks, consumer surplus and social welfare are completely characterized by the virtual output, $Q$.[16] A fee scheme is socially desirable if and only if it results in an equilibrium output that is larger than under the alternative scheme.

To compare the virtual output, consider how a marginal increase in network $j$' output (which in turn comes from a marginal reduction in either the commission rate or the unit fee) will affect its profits in the two fee schemes. For the *profit differential*, $\Pi_j^P(Q_j) - \Pi_j^U(Q_j)$, in (10), we can show that for any given $Q_j$:

[16] Given the symmetric demand (for each of the $n$ networks), $p(Q)$, and any virtual output, $Q$, social welfare on a single network is $\int_0^Q p(x)dx - \kappa_M Q$, consumer surplus is $\int_0^Q p(x)dx - Qp(Q)$, and the joint profit of the network and its affiliated merchants is $[p(Q) - \kappa_M]Q$.

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$$(11)\qquad sign\left\{\frac{\partial\Pi_j^P(Q_j)/\partial\tau_j}{\partial Q_j/\partial\tau_j}-\frac{\partial\Pi_j^U(Q_j)/\partial t_j}{\partial Q_j/\partial t_j}\right\}$$

$$= sign\left\{\underbrace{\frac{Q}{\partial Q_j/\partial\tau_j}}_{\text{Concession Effect }(-)}\underbrace{+\,\tau(\rho+2)\frac{\partial Q/\partial\tau_j}{\partial Q_j/\partial\tau_j}}_{\text{Earnings Effect }(+)}+\underbrace{2\tau(k-1)\left(1-\frac{\partial Q_{-j}/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right)}_{\text{Competition Intensification Effect }(+)}\right\},$$

where $Q$ is the virtual output, and $\tau = \tau_j$ is the symmetric commission rate corresponding to the given $Q_j$. In the expression, $[\partial\Pi_j^P(Q_j)/\partial\tau_j]/[\partial Q_j/\partial\tau_j] = 0$ and $[\partial\Pi_j^U(Q_j)/\partial t_j]/[\partial Q_j/\partial t_j] = 0$ are a network's best response functions (in term of output choices) under the two fee schemes, so $[\partial\Pi_j^P(Q_j)/\partial\tau_j]/[\partial Q_j/\partial\tau_j] - [\partial\Pi_j^U(Q_j)/\partial t_j]/[\partial Q_j/\partial t_j]$ represents the additional incentive of output expansion due to proportional fee.

As indicated by (11), marginal output expansion on a network has the following three impacts on the network's *additional profit*, which equals the commission rate times the merchants' total earnings. First, output is expanded only through a smaller commission rate, which discourages output expansion under proportional fee (relative to unit fee). This is a *concession effect*. Second, output expansion always increases merchants' total earnings,[17] which encourages output expansion under proportional fee. This is an *earnings effect*. Third, when network $j$ expands its output by lowering its commission rate or unit fee, the lower fee levels will allow its merchants to steal customers from competing networks, which will further raise the earnings of $j$'s merchants. Such effect exists under both fee schemes, but the increased merchant earnings can be partially captured by $j$ only under proportional fee. This is a *competition intensification effect*, which also encourages output expansion under proportional fee.

The decomposition in (11) is valid for an arbitrary $Q$. To compare $Q^U$ with $Q^P$ in equilibrium, we need to evaluate the sign of (11) at either $Q^U$ or $Q^P$. By making use of the two FOC's, (4) and (8), we find that unit fee is socially desirable, i.e., $Q^U > Q^P$, if and only if:

[17] To expand the output, the network has to reduce the fee level, which allows the merchants to produce a larger quantity at a smaller cost. Therefore, merchants' profits must increase with output.

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$$\varepsilon(\rho+1)+1<-\frac{(k-1)(km\varepsilon-1)}{(1-\alpha)(\rho+mk)+k}. \tag{12}$$

It must be emphasized that $\varepsilon$ and $\rho$ are endogenous and evaluated at either $Q^U$ or $Q^P$.

Given $\varepsilon \geq 1$ and $\rho > -2$, the right hand side of (12) is non-positive. Therefore, the validity of the inequality depends on the sign of $\varepsilon(\rho+1)+1$. When the demand is sub-convex (i.e., $\varepsilon(\rho+1)+1>0$) or isoelastic (i.e., $\varepsilon(\rho+1)+1=0$), (12) cannot hold, which means that proportional fee is (weakly) preferred by a social planner, with equality if and only if demand is isoelastic and networks do not compete at all (i.e., $\alpha=0$ and hence $k=1$).[18]

Now consider super-convex demands (i.e., $\varepsilon(\rho+1)+1<0$). It can be shown that the right hand side of (12) monotonically decreases with $n$ or $\alpha$. When $\alpha=0$ and hence $k=1$ (i.e., monopoly networks), (12) always holds. When $n\to\infty$ or $\alpha=1$, (12) always fails. As a result, for a given super-convex demand, fixing $n$ (or $\alpha$), there always exists a unique $\widehat{a}$ (or $\widehat{n}$) within the feasible range such that unit fee is socially preferred if and only if $\alpha<\widehat{\alpha}$ (or $n<\widehat{n}$).

Figure 2 draws the equal-welfare boundaries (i.e., welfare is equalized between the two fee schemes) in the space of $\{\rho,\varepsilon\}$.[19] Unit fee is socially more desirable than proportional fee if and only if the equilibrium point falls to the left of the corresponding equal-welfare boundary. The graph shows that a decrease in $n$ or $\alpha$ moves an equal-welfare boundary to the right, indicating an expansion of the region for socially desirable unit fee.

*Proposition 1.* When merchants single-home, unit fee is socially preferred if and only if (12) holds. In particular,

(i) if demand is sub-convex or isoelastic, then proportional fee is (weakly) preferred.

(ii) If demand is super-convex, then unit fee is preferred if and only if network competition is sufficiently weak (i.e., $n$ or $\alpha$ is smaller than some thresholds).

[18] This is established under the model's assumption that network cost ($\kappa_N$) is zero. If $\kappa_N>0$ (as shown in the appendix), social welfare would be higher under proportional fee. Both results are consistent with Shy and Wang [2011], whose equations (18) and (28), corresponding to the welfare under proportional fee and unit fee, are identical if their network cost $\nu=0$.

[19] In the $\{\rho,\varepsilon\}$ space, a particular demand function is represented by a smooth curve (see Figure 3), whereas a particular equilibrium is a point on the curve. Mrázová and Neary [2017] refer to such a curve as 'demand manifold' and suggest that 'knowing the values of the elasticity and convexity of demand that a firm faces is sufficient to predict its responses to a wide range of exogenous shocks.'

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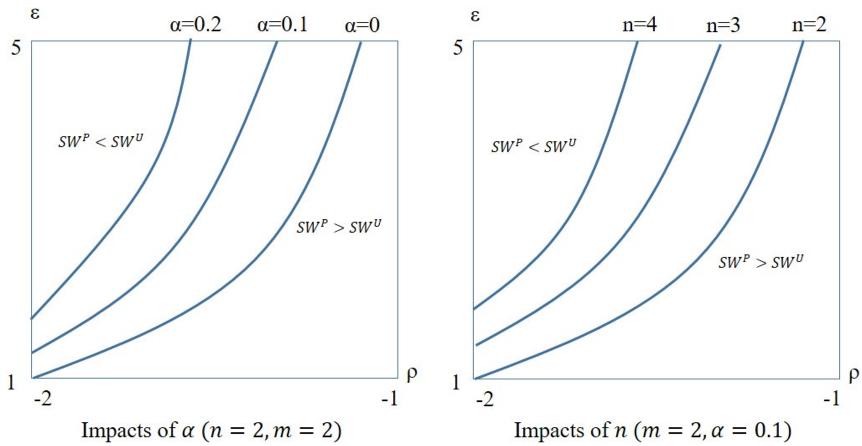


Figure 2
Single-Homing Merchants: The Equal-Welfare Boundaries

When demand is super-convex, network competition plays a crucial role for welfare comparison. The weaker the network competition, the more likely for unit fee to welfare dominate proportional fee. A super-convex demand becomes less elastic when the output shrinks (Mrázová and Neary [2017]), which will increase the network's pricing power and hence its incentive to reduce output under proportional fee. On monopoly networks ($k = 1$), this leads to an unambiguous social preference for unit fee. When networks compete ($k > 1$), the positive sign of the competition intensification effect indicates that proportional fee provides an additional incentive for each network to expand its output. In equilibrium, then, it is more likely for proportional fee to dominate unit fee.

To investigate how the welfare comparison is influenced by the parameters, we look at a class of demand functions:[20]

$$p(Q) = \psi(Q+b)^{-\beta}, \text{ with } \psi > 0,\ \beta \in (0, 1). \tag{13}$$

Then $\varepsilon = (Q+b)/(\beta Q)$, $\rho = -(\beta+1)\,Q/(Q+b)$, and the demand is super-convex if $b < 0$, sub-convex if $b > 0$, and isoelastic if $b = 0$. In addition,

$$\rho = -\frac{\beta+1}{\beta\varepsilon}. \tag{14}$$

When a parameter changes, the equilibrium output will change, but its $\rho$ and $\varepsilon$ must satisfy (14). For this reason, we will follow Mrázová and Neary

[20] Pollak [1971] has discussed this class of demand functions. Although the demand takes a special functional form, it allows for both sub-convex and super-convex cases and also nests a certain number of widely-used demand functions such as linear demand and isoelastic demands.

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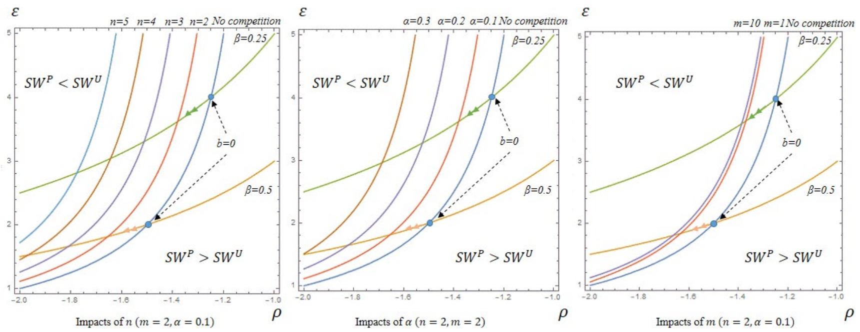


Figure 3
Single-Homing Merchants: Comparative Statics

[2017] to refer to (14) as a 'demand manifold.'[21] Note that (14) is independent of $b$. For any given $\beta$, therefore, all equilibrium outcomes stay on the same curve regardless of demand convexity.

Figure 3 shows (14) together with condition (12) in the space of $\{\rho, \varepsilon\}$. The two relatively flatter curves are the demand manifolds corresponding to $\beta = 0.25$ and $\beta = 0.5$, respectively. The intersection between the equal-welfare boundary for 'no competition' (i.e. $\varepsilon(\rho + 1) + 1 = 0$) and a demand manifold indicates the equilibrium when $b = 0$. At this point, we have $\varepsilon = \frac{1}{\beta}$ and $\rho = -(\beta + 1)$, so a change in the equilibrium output will not move its position. If $b < 0$ (i.e., super-convex demands), the equilibrium is always on the left side of the $b = 0$ point, and moves along the demand manifold to the left as the equilibrium output drops. If $b > 0$ (i.e., sub-convex demands), the equilibrium is on the right side.

Fixing all other parameters, as $\kappa_M$ increases, the equal-welfare boundaries remain unchanged (condition (12) is independent of $\kappa_M$), while the equilibrium output decreases. Therefore, the equilibrium moves from a region of $SW^P > SW^U$ to a region of $SW^P < SW^U$. Similarly, fixing all other parameters, as $n$, $\alpha$, or $m$ decreases, the equal-welfare boundary move to the right,[22] whereas the equilibrium output decreases, meaning the equilibrium point moves to the left. The opposite movements of the equilibrium point and the equal-welfare boundary again indicates that the equilibrium is more likely to result in $SW^P < SW^U$.

*Proposition 2.* Suppose merchants single-home, and the demand is $p(Q) = \psi(Q+b)^{-\beta}$ with $b < 0$ (i.e., the demand is super-convex). Then, any of the following will expand the $\varepsilon - \rho$ space on which *unit fee welfare dominates proportional fee:*

[21] All equilibrium points are on this locus, but not every point corresponds to an equilibrium. We thank an anonymous referee for pointing this out.

[22] The impact of $m$ is proved in Appendix A(ii).

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(i) the number of networks is smaller (i.e., $n$ is smaller); or
(ii) network services are less substitutable (i.e., $\alpha$ is smaller); or
(iii) the number of merchants is smaller (i.e., $m$ is smaller); or
(iv) merchant cost is larger (i.e., $\kappa_M$ is larger).

For Proposition 2, results (i) and (ii) are similar to Proposition 1 except that here we explicitly consider how a change in $n$ or $\alpha$ will move the equilibrium point in addition to the equal-welfare boundaries. Results (iii) and (iv) are about the remaining two parameters, $m$ and $\kappa_M$. Unit fee is more likely to dominate proportional fee if merchant competition weakens or their cost rises. In both cases, proportional fee brings a smaller amount of additional profit, so the competition intensification effect is weaker.

## VI. MULTI-HOMING MERCHANTS

We now turn to multi-homing merchants, focusing on the additional effects in the discount effect and welfare comparison.

### VI(i). *The Discount Effect*

From the equilibrium characterization, under unit fee, the average revenue for network $j$ is:

$$t_j = p_j + \frac{1}{m}\frac{\partial p_j}{\partial Q_j}Q_j + \frac{1}{m}\sum_{-j}\frac{\partial p_{-j}}{\partial Q_j}Q_{-j} - \kappa_M. \tag{15}$$

Under proportional fee, it is:

$$\tau_j p_j = p_j + \frac{1}{m}\left(1-\tau_j\right)\frac{\partial p_j}{\partial Q_j}Q_j + \frac{1}{m}\sum_{-j}\left(1-\tau_{-j}\right)\frac{\partial p_{-j}}{\partial Q_j}Q_{-j} - \kappa_M. \tag{16}$$

The discount effect is therefore:

$$\tau_j p_j - t_j = \underbrace{\underbrace{\left[-\frac{1}{m}\tau_j\frac{\partial p_j}{\partial Q_j}Q_j\right]}_{\text{Direct Effect }(+)} + \underbrace{\left[-\frac{1}{m}\sum_{-j}\tau_{-j}\frac{\partial p_{-j}}{\partial Q_j}Q_{-j}\right]}_{\text{Indirect Effect }(+)}}_{\text{Discount Effect }(+)}, \tag{17}$$

Merchant multi-homing brings an indirect effect on top of the direct effect seen in single-homing. As before, here we are fixing the outputs of all networks and look at how network $j$'s average revenue changes when all

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networks simultaneously switch from unit fee to proportional fee. Consider a particular merchant, $i$. When network $j$ switches to proportional fee, merchant $i$ tends to sell more through network $j$. This is the (direct) discount effect established earlier in the single-homing setting where $i$ sells only on $j$. With multi-homing, $i$ also sells its product on another network, say $j'$. Because $p_j$ tends to drop due to the direct effect, and because $i$'s products on $j$ and $j'$ are imperfect substitutes, $p_{j'}$ also tends to drop even though $Q_{j'}$ is fixed. Under proportional fee (but not under unit fee), the lower $p_{j'}$ on network $j'$ implies that $i$ will pay a lower fee to $j'$, which is an additional benefit to merchant $i$. As a result, multi-homing by merchant $i$ allows network $j$ to take away an even larger amount of profits (on top of those feasible with single-homing) from $i$ without reducing $i$'s output. This is the indirect discount effect.

### VI(ii). *Welfare Comparison*

At any given $Q_j$, network $j$'s additional profit under proportional fee can be written as:

$$\Delta\Pi_j(Q_j)=\Pi_j^P(Q_j)-\Pi_j^U(Q_j)=-\frac{1}{m}\left[\tau_j p_j' Q_j^2+\alpha\sum_{-j}\tau_{-j}p_{-j}'Q_{-j}Q_j\right].$$

We can show:

$$\text{(18)}\quad sign\left\{\frac{\partial\Pi_j^P(Q_j)/\partial\tau_j}{\partial Q_j/\partial\tau_j}-\frac{\partial\Pi_j^U(Q_j)/\partial t_j}{\partial Q_j/\partial t_j}\right\}$$

$$= sign\left\{\begin{array}{l}\underbrace{\frac{Q}{\partial Q_j/\partial\tau_j}}_{\text{Concession Effect }(-)}+\underbrace{\tau(\rho+2)\frac{\partial Q/\partial\tau_j}{\partial Q_j/\partial\tau_j}}_{\text{Direct Earnings Effect }(+)}+\underbrace{2\tau(k-1)\left(1-\frac{\partial Q_{-j}/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right)}_{\text{Competition intensification Effect }(+)}\\ \underbrace{+\tau(k-1)\left[(\rho+2)\left(\alpha+(k-\alpha)\frac{\partial Q_{-j}/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right)+(k-2\alpha)\left(1-\frac{\partial Q_{-j}/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right)\right]}_{\text{Indirect Earnings Effect }(+)}\\ +\underbrace{\left\{(k-1)\left[(k+1-\alpha)\rho+(m+1)k\right]\left(\frac{\partial Q_{-j}/\partial t_j}{\partial Q_j/\partial t_j}-\frac{\partial Q_{-j}/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right)\right\}}_{\text{Allocation Effect }(+)}\end{array}\right\},$$

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where $\tau = \tau_j$ is the symmetric commission rate corresponding to the output levels.

As can be seen from the decomposition, when merchants multi-home, proportional fee affects the equilibrium output through five forces. The first three, namely the (direct) earnings effect, concession effect, and competition intensification effect, are the same as when merchants single-home. Multi-homing gives rise to two additional effects. When a network lowers the fee to its merchants, these merchants will reap some benefits from their products sold on other networks, and these benefits can then be turned into additional profits for the network that is lowering the fee. This is an *indirect earnings effect*, which gives every network an additional incentive to expand under proportional fee.

When merchants single-home, networks compete indirectly through merchants' output choices. When merchants multi-home, in addition to the indirect, quantity competition through merchants' output levels, there is also a direct, price competition between networks given that each merchant also reallocates its total transactions in favor of a network that charges a lower fee. This again tends to improve merchants' profit, part of which will be turned into a network's profit under proportional fee. Therefore, proportional fee encourages each network to further expand. This is an *allocation effect*.

For the total effect, we find that unit fee welfare dominates proportional fee, i.e., $Q^U > Q^P$, if and only if (evaluated at $Q^U$ or $Q^P$):

$$\varepsilon\,(\rho+1)+1 < -\frac{\varepsilon\,(k-1)\,(\rho+m+1)\,(k-\alpha+1)}{(1-\alpha)\,[(1-\alpha)\,\rho+m+1]}. \tag{19}$$

When $\alpha = 0$, the setting degenerates into monopoly networks, and indeed (19) would be identical to (12). That is, when networks do not compete, multi-homing does not make any difference, which is what we should expect.

Given $\varepsilon \geq 1$ and $\rho > -2$, the right hand side of (19) is non-positive. Similar to the single-homing case, when demand is sub-convex or isoelastic, (19) never holds, meaning that proportional fee is (weakly) preferred, with equality if and only if demand is isoelastic and networks are monopolies (i.e., $k = 1$).

When demand is super-convex, the left hand side of (19) is also negative. It can be shown that the right hand side of (19) decreases with $n$ and $\alpha$. When $\alpha = 0$ (i.e., monopoly networks), (19) always holds. When $n \to \infty$ or $\alpha = 1$, (19) never holds. As a result, for a given super-convex demand, fixing $n$ (or $\alpha$), there always exists a unique $\widetilde{\alpha}$ (or $\widetilde{n}$) within the feasible range such that unit fee socially dominates proportional fee if and only if $\alpha < \widetilde{\alpha}$ (or $n < \widetilde{n}$). That is, unit fee is more likely to dominate proportional fee when network competition weakens. It is now clear that welfare comparison between the two fee schemes is qualitatively similar whether

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merchants single or multi-home. *When merchants multi-home, if demand is sub-convex or isoelastic, proportional fee is (weakly) preferred; if demand is super-convex, then unit fee is preferred if and only if network competition is sufficiently weak.*

For the impacts of the underlying parameters, we again look at demand functions (13) and therefore $\rho = -\frac{\beta+1}{\beta\varepsilon}$, which is not affected by multi-homing. Then Proposition 2 continues to hold. *It is more likely for unit fee to welfare dominate proportional fee if the number of network is smaller, network services are less substitutable, the number of merchant is smaller, or merchant cost is larger.*

The difference between conditions (19) and (12) captures the effects of multi-homing. For arbitrary $(\rho, \varepsilon)$, we can show that multi-homing strengthens the social preference for proportional fee. Fixing all parameters, the equal-welfare boundary for multi-homing is always on the left side of the corresponding boundary for single-homing. This can be seen from Figure 4 for the demand class (13), where arrows on the demand manifolds indicate how the equilibrium moves when equilibrium quantity drops.

Fixing a fee scheme, moving from single-homing to multi-homing may raise or lower the equilibrium output.[23] Nevertheless, when network differentiation is small enough (i.e., $\alpha$ is large enough), multi-homing will generate a larger equilibrium output, as double marginalization tends to be eliminated under multi-homing but not single-homing if $\alpha \to 1$. Therefore, when merchants switch from multi-homing to single-homing, the equilibrium output decreases, and thus the equilibrium point moves along the demand manifolds from regions of $SW^P > SW^U$ to $SW^P < SW^U$. At the same time, the equal-welfare boundary moves from left to right. The opposite movements indicate that unit fee is more likely to welfare dominate proportional fee.

*Proposition 3.* Suppose the demand is $p(Q) = \psi(Q+b)^{-\beta}$ with $b < 0$ (i.e., the demand is super-convex) and networks differentiation is small. Then merchant multi-homing shrinks the $\varepsilon - \rho$ space on which unit fee welfare dominates proportional fee.

The driving force for multi-homing to favor proportional fee is the indirect discount effect in (17) as well as the allocation and indirect earnings effects

[23] Multi-homing can be regarded as a merger among single-homing merchants, and a merged merchant naturally internalizes the negative externalities between its outputs on different networks. Fixing the network fee levels, such a coordination effect tends to reduce equilibrium output. However, each merchant will also reallocate its transactions in favor of the network that charges the lowest fee. Such an allocation effect (as established earlier) will intensify network competition, resulting in lower network fees and hence tends to raise the equilibrium output. Either effect may dominate, so multi-homing may strengthen or weaken the overall competition in both unit fee and proportional fee schemes.

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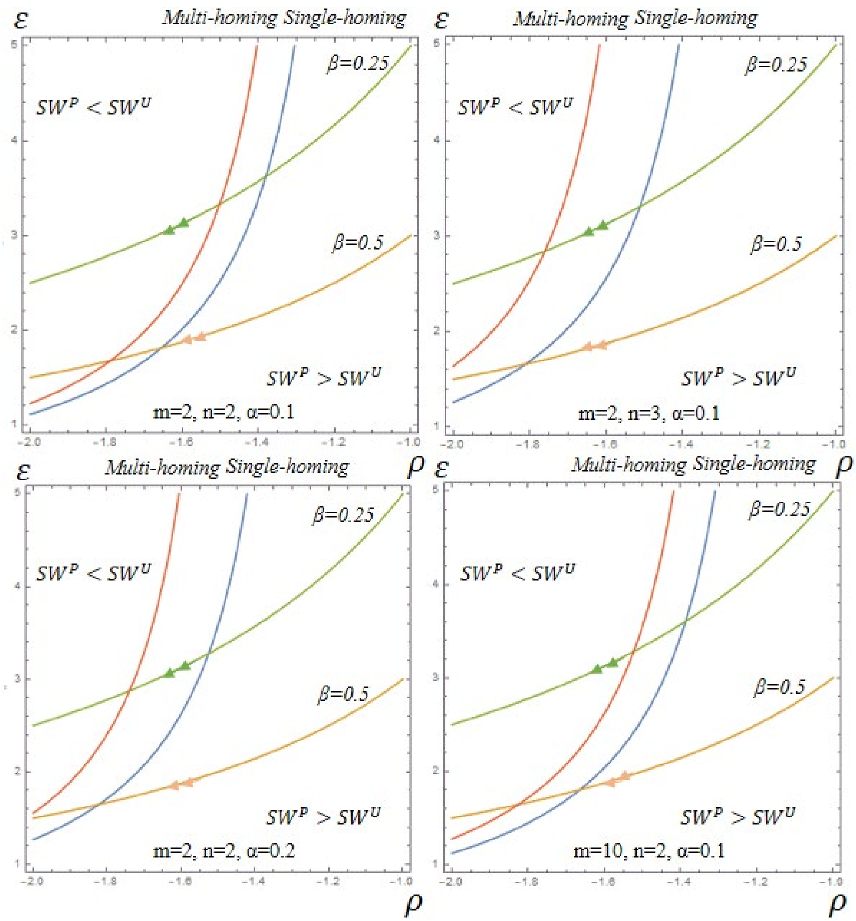


Figure 4
Multi-Homing versus Single-Homing

in (18). Both encourage each network to further expand its output under proportional fee when merchants multi-home.

## VII. PROFITS

So far we have been comparing the two fee schemes for their welfare consequences, which is the focus of this paper. Different fee schemes will also lead to different profits for networks and merchants, which is analyzed now. The profit comparison is useful because it sheds light on which fee scheme will be adopted in equilibrium. To save notations, we will use $Q^U$ and $Q^P$ to denote the equilibrium outputs under both single-homing and multi-homing, and will indicate which is the case when necessary.

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When merchants single-home, the total profits of all merchants and those of all networks are (the subscripts $M$ and $N$ indicate merchants and networks respectively):

$$
\begin{aligned}
&\Pi_M^U(Q^U) = -\frac{n}{mk^2}p'(Q^U)(Q^U)^2;\\
&\Pi_N^U(Q^U) = \frac{n}{k}[p(Q^U) - \kappa_M]Q^U + \frac{n}{mk^2}p'(Q^U)(Q^U)^2.\\
&\Pi_M^P(Q^P) = -\left[1-\tau(Q^P)\right]\frac{n}{mk^2}p'(Q^P)(Q^P)^2;\\
&\Pi_N^P(Q^P) = \frac{n}{k}[p(Q^P) - \kappa_M]Q^P + \left[1-\tau(Q^P)\right]\frac{n}{mk^2}p'(Q^P)(Q^P)^2.
\end{aligned}
$$

When merchants multi-home, these profits are:

$$
\begin{aligned}
&\Pi_M^U(Q^U) = -\frac{n}{mk}p'(Q^U)(Q^U)^2;\\
&\Pi_N^U(Q^U) = [p(Q^U) - \kappa_M]Q^U + \frac{n}{mk}p'(Q^U)(Q^U)^2.\\
&\Pi_M^P(Q^P) = -\left[1-\tau(Q^P)\right]\frac{n}{mk}p'(Q^P)(Q^P)^2;\\
&\Pi_N^P(Q^P) = [p(Q^P) - \kappa_M]Q^P + \left[1-\tau(Q^P)\right]\frac{n}{mk}p'(Q^P)(Q^P)^2.
\end{aligned}
$$

For any given $Q$, multi-homing increases merchant profit due to their ability to allocate transactions across networks, which in turn will reduce the network profit, as the two parties' joint profit is invariant to the redistribution (i.e., $\Pi_M^U(Q) + \Pi_N^U(Q) = \Pi_M^P(Q) + \Pi_N^P(Q)$). Nevertheless, the profit functions under the two homing arrangements have similar properties. For any given fee scheme, merchant profit increases with $Q$, whereas network profit decreases with $Q$ when $Q$ exceeds the equilibrium level. Fixing $Q$, proportional fee benefits networks at the expense of merchants, i.e., $\Pi_M^P(Q) < \Pi_M^U(Q)$ and $\Pi_N^P(Q) > \Pi_N^U(Q)$).

Recall that welfare comparison depends crucially on the intensity of network competition. The same is true for profit comparison. In what follows, we shall focus on two scenarios: network competition is weak (i.e., $\alpha \to 0$) or strong (i.e., $\alpha \to 1$). In addition, we shall also analyze networks' individual and independent incentives in adopting a particular fee type.

### VII(i). *Weak Network Competition*

When $\alpha \to 0$, network competition is weak, and the networks can be viewed as $n$ identical monopolies. In that case, profit comparison for the network is straightforward and unambiguous:

$$\Pi_N^P(Q^P) \geq \Pi_N^P(Q^U) > \Pi_N^U(Q^U).$$

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The first inequality comes from revealed preference, and the second inequality is due to the discount effect. Therefore, networks always prefer proportional fee. This is a robust result independent of demand properties.

Now turn to the merchants. Since merchant profit always increases with the total output given a fee scheme, when the networks switch from unit fee to proportional fee, merchant profit changes through two channels. First, fixing $Q^U$, networks gain, implying that merchants lose. Second, moving from the equilibrium $Q^U$ to the equilibrium $Q^P$, merchants may gain or lose depending on whether the output increases or decreases. If $Q^P \leq Q^U$, the two effects move in the same direction for merchants, who are unambiguously worse off under proportional fee. If $Q^P > Q^U$, the two effects move in opposite directions and the impact on merchants is ambiguous. To summarize,

*Proposition 4.* When network competition is weak (i.e., $\alpha \to 0$), a switch from unit fee to proportional fee has the following consequences.

(i) Networks always gain.
(ii) If consumers gain (i.e. $Q^P > Q^U$), then merchants may gain or lose; if consumers lose (i.e., $Q^P \leq Q^U$), then merchants also lose.

The following Table I summarizes the comparison for each party involved.

Among the three parties, networks are the most enthusiastic about proportional fee, followed by consumers, and merchants are the least enthusiastic. If demand is super-convex, proportional fee benefits networks at the expense of social welfare. Because consumer surplus is fully aligned with social welfare, consumers' complaints about proportional fee is a strong signal that proportional fee is hurting social welfare. In that case, networks should be forced to abandon proportional fee. If consumers are hurt by proportional fee, then merchants must also be hurt, and merchants' complaints about proportional fee should not be ignored, as it may indicate a damage to social welfare.

### VII(ii). *Strong Network Competition*

When $\alpha \to 1$, network competition is strong, and the equilibrium outputs in both fee schemes can exceed $Q^*$, the output level that would maximize the following joint profit of networks and merchants:[24]

[24] The only exception is that merchants single-home and the number of merchants or networks is extremely small (i.e., $m = 1$ and $n = 2$), which lead to strong double markup. See Gu *et al*. [2019] for details.

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TABLE I
IMPACTS OF SWITCHING FROM UNIT FEE TO PROPORTIONAL FEE

| Demand | networks | consumer | merchants | welfare |
|---|---|---|---|---|
| Sub-convex | ↑ | ↑ | ↑ or ↓ | ↑ |
| Isoelastic | ↑ | = | ↓ | = |
| Super-convex | ↑ | ↓ | ↓ | ↓ |

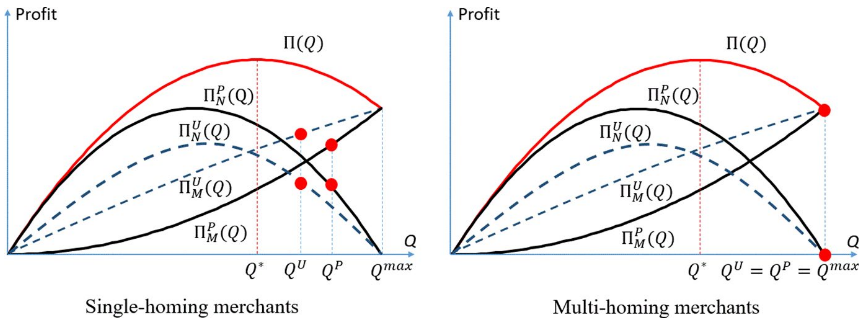


Figure 5
Profits when Network Competition Is Strong ($\alpha \to 1$)

$$\Pi(Q) = [p(Q) - \kappa_M]Q.$$

Therefore, a further increase of the output will reduce the supply chain's joint profit.

If merchants single-home, then unit fee will allow networks to maintain positive markups and profits even though $\alpha \to 1$. This, in turn, leaves room for proportional fee to push the equilibrium further away from joint profit maximization (i.e., $Q^* < Q^U < Q^P$), so the supply chain's joint profit must drop. This means that networks and merchants cannot both gain. The impact on either party is ambiguous, so each may gain or lose. An example of profit comparison is illustrated in the left panel of Figure 5.

If merchants multi-home, given that network services are almost perfect substitutes ($\alpha \to 1$), every merchant will concentrate all its transactions on the network with the lowest fee. Such a Bertrand type of network competition drives down the equilibrium fee to networks' marginal cost, and all networks have zero markup and profits. This is true in both fee schemes, leading to exactly the same equilibrium outcome.[25] The right panel of Figure 5 illustrates the situation under multi-homing.

*Proposition 5.* When network competition is strong (i.e., $\alpha \to 1$), a switch from unit fee to proportional fee has the following consequences.

[25] When $\alpha \to 1$, the equilibrium is a corner solution, and condition (19) no longer applies.

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(i) If merchants single-home, the supply chain's joint profit drops; networks and merchants may each gain or lose, but cannot both gain.
(ii) If merchants multi-home, there is no change in either party's profit. In both fee schemes, network profit is zero, while merchant profit is positive.

Proposition 5(i) highlights the possibility for networks to prefer unit fee and merchants to prefer proportional fee. This is the opposite of Johnson's [2017] finding that networks would prefer proportional fee while merchants prefer unit fee. The different conclusions are driven by different ways of modeling network competition. Johnson [2017] adopts a constant conduct-parameter approach by assuming that a player's conduct is dictated by an exogenous parameter that is invariant to the fee scheme. By contrast, we have taken a standard approach of successive oligopoly, which fully endogenizes firms' behavior as well as competition intensity. This turns out to be crucial, as the analysis shows that proportional fee intensifies network competition, which would benefit merchants but hurt networks.

Our result also suggests that, when merchants single-home and network services are close substitutes, networks and merchants will have conflicting interests in choosing the fee type: whenever one party is better off, the other must be worse off. When merchants multi-home and network services are close substitutes, the choice of the fee scheme no longer matters for either networks or merchants.

### VII(iii). *Individual Preferences for the Fee Type*

So far we have assumed that the same fee scheme is used by all competing parties, so a larger profit for either networks or merchants indicates the party's collective preference. This assumption helps to simplify the analysis given that the demand is very general. In what follows, we shall explore individual preferences by allowing different fee schemes by different networks. Consider two networks ($n=2$), each with one single-homing merchant ($m=1$). The demand for network $i \in \{1,2\}$ is:

$$q_i = 1 - p_i + p_j, \text{ with } j \neq i. \tag{20}$$

Such a linear inelastic demand can be derived from a standard Hotelling model. Because each network can adopt either fee scheme independently, there are three possible combinations:

1. Both adopt unit fee, then a network's equilibrium profit is $\Pi_N^{UU}(\kappa_M) = 3$, and merchant profit is $\Pi_M^{UU}(\kappa_M) = 1$.

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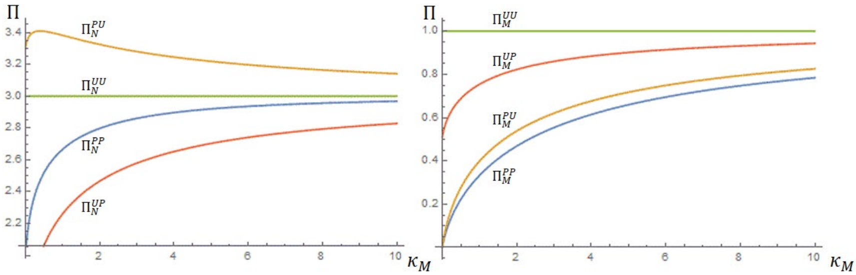


Figure 6
Individual Profitabilities: Networks (left panel) and Merchants (right panel)

2. Both adopt proportional fee, then the equilibrium profits depend on the only remaining parameter, $\kappa_M$, and are denoted as $\Pi_N^{PP}(\kappa_M)$ and $\Pi_M^{PP}(\kappa_M)$ for network and merchant, respectively.[26]
3. One adopts proportional fee and the other adopts unit fee. The profits are denoted by $\Pi_N^{PU}(\kappa_M)$, $\Pi_N^{UP}(\kappa_M)$, $\Pi_M^{PU}(\kappa_M)$, and $\Pi_M^{UP}(\kappa_M)$.[27]

Figure 6 plots these profit curves as functions of $\kappa_M$, where the left panel shows network profit, and the right panel shows merchant profit. For networks,

$$\Pi_N^{PU} > \Pi_N^{UU} > \Pi_N^{PP} > \Pi_N^{UP}$$

for any given $\kappa_M$, which indicates that proportional fee is each network's dominant strategy, but the networks are collectively worse off. In other words, this is a Prisoners' Dilemma. For merchants,

$$\Pi_M^{UU} > \Pi_M^{UP} > \Pi_M^{PU} > \Pi_M^{PP},$$

which indicates that merchants always prefer unit fee, both individually and collectively.

*Proposition 6.* Suppose there are two networks and each has one single-homing merchant, and the demand is (20).

[26] The expressions of the profit functions can be found in Appendix A(v).
[27] In the notation, the superscript indicates the combination of the fee schemes, where the first letter is a party's own fee type, and the second letter is its competitor's type.

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(i) For networks, proportional fee is a dominant strategy, but networks are collectively worse off under proportional fee.
(ii) For merchants, unit fee is a dominant strategy, and merchants are also collectively better off under unit fee.

Proportional fee turns out to be each network's dominant strategy. When a network switches unilaterally from unit fee to proportional fee, the discount effect encourages its merchant to be more aggressive in stealing businesses from the competing network's merchants, which increases the network's profit. However, since the joint demand is inelastic, when both networks adopt proportional fee, the intensified competition reduces both profits without raising the total output. This leads to a Prisoners' Dilemma for the two networks.[28] In fact, the dilemma hurts not only networks, but also merchants, who are also encouraged by their respective networks to compete more aggressively under proportional fees.

Proposition 6 can be combined with Proposition 4 to offer an explanation of why proportional fee is so common nowadays, especially in the digital economy. Facing no competition, a network always prefers proportional fee because it leads to a higher profit. Facing competition, a network also prefers proportional fee because it is a dominant strategy even though the network may end up worse off due to prisoners' dilemma. In both cases, proportional fee reduces merchants' profits. This may explain the widespread complaints about proportional fee by merchants.

Another interesting property to notice is that a merchant's profit increases with its own cost when at least one network adopts proportional fee ($\Pi_M^{UP}$, $\Pi_M^{PU}$ and $\Pi_M^{PP}$ all increases with $\kappa_M$). This is because a smaller merchant cost means a smaller distortion due to double marginalization, which allows a network to levy a higher commission rate, and therefore a smaller profit left for the merchant.

## VIII. CONCLUSION

This paper compares social welfare between proportional and unit fee in a setting with competing networks and general demands (with a mild restriction of symmetric differentiation across networks). When demand is sub-convex or isoelastic, a social planner (weakly) prefers proportional fee, and the preference is independent of network competition. If demand is super-convex, unit fee may well dominate proportional fee, and we identify the sufficient and necessary conditions for this to happen. Both network competition and merchant

[28] In a similar setting, Aiura and Ogawa [2013] reached similar conclusions when two governments compete for cross-border shoppers by choosing between ad valorem and unit (specific) taxes. We went further by also identifying merchants' preferences for the fee types.

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homing play crucial roles in such a comparison. In particular, unit fee is more likely to be socially desirable if network competition is weak and merchants must single-home. Therefore, the antitrust authority should pay special attentions to such a setting and, if necessary, force a change of the fee scheme.

The research can be extended in several directions. First, although a network can collect fees from both sides of the market, price neutrality implies that the two-sided setting is equivalent to a vertical market structure of successive oligopolies. Given the increasing presence of two-sided markets in the digital economy and the associated theoretical interests, it is important to extend our model to a truly two-sided market where price neutrality no longer holds.[29] Second, the conclusion that proportional fee is networks' dominant strategy is established for a specific demand. We suspect the result holds for more general demands, but the analysis is challenging and is therefore left for future work.

## APPENDIX

In order to focus on the key messages, the main text has presented the results for a simpler case of zero network cost ($\kappa_N = 0$). Here in the appendix we deal with the general case of $\kappa_N \geq 0$. In what follows, $p'$ and $p''$ are the first and second derivatives of the demand function, respectively.

### A(i). *Single-Homing: Equilibrium Characterization*

We characterize the equilibrium in the two fee schemes respectively.

*Unit fee*

Take derivative of (1) with respect to $t_j$ and $t_{-j}$. Since $\frac{\partial Q_{-j}}{\partial t_{-j}} = \frac{\partial Q_j}{\partial t_j}$ and $\frac{\partial Q_{-j}}{\partial t_j} = \frac{\partial Q_j}{\partial t_{-j}}$, we have

$$\left[\rho+(m+1)k\right]\frac{\partial Q_j}{\partial t_j}+(k-1)(\rho+mk)\frac{\partial Q_{-j}}{\partial t_j}=\frac{mk}{p'};$$
$$\alpha(\rho+mk)\frac{\partial Q_j}{\partial t_j}+\left\{(k-\alpha)\rho+k\left[1+m(k-\alpha)\right]\right\}\frac{\partial Q_{-j}}{\partial t_j}=0.$$

Solving this equations system, we have

$$\frac{\partial Q_j}{\partial t_j}=-\frac{Q}{p}\frac{m\varepsilon\left[(k-\alpha)(\rho+mk)+k\right]}{(\rho+mk+1)\left[(1-a)(\rho+mk)+k\right]};$$
$$\frac{\partial Q_{-j}}{\partial t_j}=\frac{Q}{p}\frac{\alpha m\varepsilon(\rho+mk)}{(\rho+mk+1)\left[(1-a)(\rho+mk)+k\right]}.$$

To ensure proper behavior of network choices, we need $\frac{\partial Q_j}{\partial t_j} < 0$ and $\frac{\partial Q_{-j}}{\partial t_j} > 0$, which is guaranteed if $\rho + mk > 0$.

[29] See Armstrong [2006], Rochet and Tirole [2003, 2006], and Rysman [2009] for models of two-sided markets in which price neutrality does not hold.

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Plug $\frac{\partial Q_j}{\partial t_j}$ into $\frac{\partial \Pi_j^U(Q_j)}{\partial t_j} = Q_j + (t_j - \kappa_N)\frac{\partial Q_j}{\partial t_j} = 0$ or, equivalently, $(t_j - \kappa_N) + \frac{Q_j}{\partial Q_j/\partial t_j} = 0$. Finally, substitute $t_j = \frac{1}{mk}p'Q + p - \kappa_M$ and $Q_j = \frac{Q}{k}$ to arrive at the equilibrium characterization:

$$\kappa_M + \kappa_N = p + \frac{1}{mk}p'Q + p'Q\frac{(\rho+mk+1)\left[(1-a)(\rho+mk)+k\right]}{mk\left[(k-\alpha)(\rho+mk)+k\right]}, \tag{21}$$

which will determine a unique equilibrium total output, $Q^U$. Note that $p$, $p'$, $p''$ are all functions of $Q$.

*Proportional fee*

Take derivative of (5) with respect to $\tau_j$ and $\tau_{-j}$. Since $\frac{\partial Q_{-j}}{\partial \tau_{-j}} = \frac{\partial Q_j}{\partial \tau_j}$ and $\frac{\partial Q_{-j}}{\partial \tau_j} = \frac{\partial Q_j}{\partial \tau_{-j}}$, we have

$$\begin{aligned}
\left[\rho+(m+1)k\right]\frac{\partial Q_j}{\partial t_j} + (k-1)(\rho+mk)\frac{\partial Q_{-j}}{\partial t_j} &= \frac{mk}{p'}\frac{\kappa_M}{(1-\tau)^2};\\
\alpha(\rho+mk)\frac{\partial Q_j}{\partial t_j} + \left\{(k-\alpha)\rho + k\left[1+m(k-\alpha)\right]\right\}\frac{\partial Q_{-j}}{\partial t_j} &= 0.
\end{aligned}$$

Solving this equations system, we have

$$\begin{aligned}
\frac{\partial Q_j}{\partial \tau_j} &= -\frac{Q\kappa_M}{p(1-\tau)^2}\frac{m\left[(k-\alpha)(\rho+mk)+k\right]}{(\rho+mk+1)\left[(1-a)(\rho+mk)+k\right]};\\
\frac{\partial Q_{-j}}{\partial \tau_j} &= \frac{Q\kappa_M}{p(1-\tau)^2}\frac{\alpha m(\rho+mk)}{(\rho+mk+1)\left[(1-a)(\rho+mk)+k\right]}.
\end{aligned}$$

To ensure proper behavior of network choices, we need $\frac{\partial Q_j}{\partial \tau_j} < 0$ and $\frac{\partial Q_{-j}}{\partial \tau_j} > 0$, which is again guaranteed if $\rho + mk > 0$.

Plug $\frac{\partial Q_j}{\partial \tau_j}$ and $\frac{\partial Q_{-j}}{\partial \tau_j}$ into $\frac{\partial \Pi_j^P(Q_j)}{\partial \tau_j} = \left\{p_j + \tau_j p_j'\left[\frac{\partial Q_j}{\partial \tau_j} + (k-1)\frac{\partial Q_{-j}}{\partial \tau_j}\right]\right\}\frac{Q}{k} + (\tau_j p_j - \kappa_N)\frac{\partial Q_j}{\partial \tau_j} = 0$. Finally, substitute $\tau_j = 1 - \frac{mk}{p'Q+mkp}\kappa_M$ and $Q_j = \frac{Q}{k}$ to arrive at the equilibrium characterization:

$$\begin{aligned}
\kappa_M + \kappa_N = {} & p - \frac{\kappa_M}{km\varepsilon - 1} + p'Q\frac{(1-a)(\rho+mk)+1}{(k-\alpha)(\rho+mk)+k}\\
& -\kappa_M\frac{mk\left\{(1-\alpha)(\rho+mk)\left[km\varepsilon - 1 - \varepsilon^2(\rho+m+1)\right] + k\varepsilon\left[m-\varepsilon(\rho+m+1)\right] - 1\right\}}{(km\varepsilon-1)^2\left[(k-\alpha)(\rho+mk)+k\right]},
\end{aligned} \tag{22}$$

which will determine the unique equilibrium output under proportional fee, $Q^P$.

## A(ii). *Single-Homing: Welfare*

We can write the profit function $\Pi_j^P = \Pi_j^U + \Delta\Pi_j$. These profits ($\Pi_j^P$, $\Pi_j^U$ and $\Delta\Pi_j$) are functions of $Q_j$ and $Q_{-j}$, which are in turn functions of $\mathbf{t}$ or $\tau$.

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The decomposition is:

$$
\begin{aligned}
& sign\left\{\frac{\partial\Pi_j^P/\partial\tau_j}{\partial Q_j/\partial\tau_j}-\frac{\partial\Pi_j^U/\partial t_j}{\partial Q_j/\partial t_j}\right\} \\
= \; & sign\left\{\frac{\partial\left(\Pi_j^U+\Delta\Pi_j\right)/\partial\tau_j}{\partial Q_j/\partial\tau_j}-\frac{\partial\Pi_j^U/\partial t_j}{\partial Q_j/\partial t_j}\right\}=sign\left\{\frac{\partial\Delta\Pi_j/\partial\tau_j}{\partial Q_j/\partial\tau_j}+\frac{\partial\Pi_j^U/\partial\tau_j}{\partial Q_j/\partial\tau_j}-\frac{\partial\Pi_j^U/\partial t_j}{\partial Q_j/\partial t_j}\right\} \\
= \; & sign\left\{\frac{\partial\Delta\Pi_j/\partial\tau_j}{\partial Q_j/\partial\tau_j}+\frac{(n-1)\frac{\partial\Pi_j^U}{\partial Q_{-j}}\frac{\partial Q_{-j}}{\partial\tau_j}+\frac{\partial\Pi_j^U}{\partial Q_j}\frac{\partial Q_j}{\partial\tau_j}}{\partial Q_j/\partial\tau_j}-\frac{(n-1)\frac{\partial\Pi_j^U}{\partial Q_{-j}}\frac{\partial Q_{-j}}{\partial t_j}+\frac{\partial\Pi_j^U}{\partial Q_j}\frac{\partial Q_j}{\partial t_j}}{\partial Q_j/\partial t_j}\right\} \\
= \; & sign\left\{\frac{\partial\Delta\Pi_j/\partial\tau_j}{\partial Q_j/\partial\tau_j}+(n-1)\frac{\partial\Pi_j^U}{\partial Q_{-j}}\left(\frac{\partial Q_{-j}/\partial\tau_j}{\partial Q_j/\partial\tau_j}-\frac{\partial Q_{-j}/\partial t_j}{\partial Q_j/\partial t_j}\right)\right\} \\
= \; & sign\left\{\frac{\partial\Delta\Pi_j/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right\}=sign\left\{\frac{\partial\left[-\tau_j\frac{1}{m}p_j'Q_j^2\right]/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right\} \\
= \; & sign\left\{\underbrace{\tau(\rho+2)\frac{\partial Q/\partial\tau_j}{\partial Q_j/\partial\tau_j}}_{\text{Earnings effect (+)}}+\underbrace{\frac{Q}{\partial Q_j/\partial\tau_j}}_{\text{Concession Effect (−)}}+\underbrace{2\tau(k-1)\left(1-\frac{\partial Q_{-j}/\partial\tau_j}{\partial Q_j/\partial\tau_j}\right)}_{\text{Competition intensification Effect (+)}}\right\}.
\end{aligned}
$$

By comparing (21) with (22), we can verify that $Q^P < Q^U$ if and only if

$$\kappa_N < -\Psi\times\left\{[\varepsilon(\rho+1)+1]+\frac{(k-1)(km\varepsilon-1)}{(1-\alpha)(\rho+mk)+k}\right\}\equiv\overline{\kappa}_N,$$

where $\Psi=\frac{p(Q^U)(\rho+mk+1)\left[(1-\alpha)(\rho+mk)+k\right]^2}{\varepsilon\left[(k-\alpha)(\rho+mk)+k\right]\left\{\{mk[\varepsilon(\rho+2)+1]-1\}\left[(1-\alpha)(\rho+mk)+1\right]+(k-1)(2km\varepsilon-1)(\rho+mk+1)\right\}}>0.$
The condition becomes (12) in the main text given $\kappa_N = 0$.

*Comparative statics form*

$\frac{\partial\frac{(k-1)(km\varepsilon-1)}{(1-\alpha)(\rho+mk)+k}}{\partial m}>0$ if and only if $\rho > -\frac{k\varepsilon+(1-\alpha)}{\varepsilon(1-\alpha)}=\overline{\rho}$. Let $\widehat{\rho}(m)$ solves condition (12), i.e., $Q^P < Q^U$ if and only if $\rho<\widehat{\rho}(m)$. This means that $\widehat{\rho}(m)$ decreases with $m$ if $\widehat{\rho}(m)>\overline{\rho}$, and increases with $m$ if $\widehat{\rho}(m)<\overline{\rho}$.

Take limitation of $m$ and solve the condition (12), we have $Q^P < Q^U$ if and only if $\rho < -\frac{(k-a)\varepsilon+(1-\alpha)}{\varepsilon(1-\alpha)}=\widehat{\rho}$. Obviously, $\widehat{\rho}>\overline{\rho}$. Suppose there is $\overline{m}$ such that $\widehat{\rho}(\overline{m})<\widehat{\rho}$. If $\widehat{\rho}(\overline{m})>\overline{\rho}$, then $\widehat{\rho}(m)$ decreases with $m$ for $m>\overline{m}$. However, $\widehat{\rho}(\overline{m})<\widehat{\rho}$ indicates $m>\infty$, which is obviously impossible. If $\widehat{\rho}(\overline{m})\le\overline{\rho}$, then there must exist a sufficiently large $m>\overline{m}$ such that $\widehat{\rho}(m)>\overline{\rho}$. Again, it results in the same contradiction. Therefore, we always have $\widehat{\rho}(m)>\widehat{\rho}>\overline{\rho}$, and thus $\widehat{\rho}(m)$ decreases with $m$. Or equivalently, a larger $m$ makes condition (12) harder to sustain.

## A(iii). *Multi-Homing: Equilibrium Characterization*

Again the equilibrium is characterized for the two fee schemes respectively.

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*Unit fee*

Merchant $i \in \{1, 2, \cdots, m\}$ sells on all $n$ networks, so its profit under unit fee is $\pi_i^U = \sum_{j=1}^n [p_j - t_j - \kappa_M] q_{ij}$. Merchant FOC, $\frac{\partial \pi_i^U}{\partial q_{ij}} = 0$, determines the quantity sold on network $j \in \{1, 2, \cdots, n\}$. Summing up the FOC's over $i$, we have the average revenue for network $j$:

$$t_j = p_j + \frac{1}{m} \frac{\partial p_j}{\partial Q_j} Q_j + \frac{1}{m} \sum_{-j} \frac{\partial p_{-j}}{\partial Q_j} Q_{-j} - \kappa_M, \tag{23}$$

where $Q_j = \sum_{i=1}^m q_{ij}$ is the total output sold on network $j$, and $p_{-j}$ is the price on a network other than $j$. The system of merchant FOC's (there are $n$ of them similar to (23)) will collectively determine $Q_j\left(t_j, \mathbf{t}_{-j}\right)$, $j \in \{1, .., n\}$, where $\mathbf{t}_{-j}$ is the vector of other networks' unit fee.

Network $j$'s profit is $\Pi_j^U\left(t_j\right) = (t_j - \kappa_N) Q_j\left(t_j, \mathbf{t}_{-j}\right)$. Then network FOC's for all the $n$ networks will collectively determine a unique equilibrium in terms of the $n$ unit fee. In equilibrium, we have $Q_j^U = Q_{-j}^U$, and the virtual output, $Q^U = kQ_j^U$.

To characterize the equilibrium, we need to substitute $\frac{\partial Q_j}{\partial t_j}$ into networks' FOC's. In what follows, we will show how to obtain $\frac{\partial Q_j}{\partial t_j}$. From merchants' FOC's and combine them together, we have

$$\begin{aligned} t_j &= p_j + \frac{1}{m} p_j' Q_j + \frac{\alpha}{m} \sum_{-j} p_{-j}' Q_{-j} - \kappa_M; \\ \sum_{j=1}^n t_j &= \sum_{j=1}^n p_j + \frac{k}{m} \sum_{j=1}^n p_j' Q_j - n\kappa_M. \end{aligned}$$

By taking derivative with respect to $t_j$ on both sides of the equations, we obtain an equation system, from which we can calculate $\frac{\partial Q_j}{\partial t_j}$ and $\frac{\partial Q}{\partial t_j} = \frac{\partial Q_j}{\partial t_j} + (k-1) \frac{\partial Q_{-j}}{\partial t_j}$. Then, we have

$$\begin{aligned} \frac{\partial Q_j}{\partial t_j} &= \frac{m \{k (k-\alpha)(\rho+m+1) - \alpha (1-\alpha) \rho\}}{p' (1-\alpha)(\rho+m+1) k \left[(1-\alpha)\rho + k(m+1)\right]}; \\ \frac{\partial Q_{-j}}{\partial t_j} &= -\frac{\alpha m \{k(\rho+m+1) + (1-\alpha)\rho\}}{p'(1-\alpha)(\rho+m+1) k \left[(1-\alpha)\rho + k(m+1)\right]}; \\ t &= p + \tfrac{1}{m} p' Q - \kappa_M. \end{aligned}$$

To ensure the equilibrium is well-behaved, we need $\frac{\partial Q_j}{\partial t_j} < 0$ and $\frac{\partial Q_{-j}}{\partial t_j} > 0$, which is guaranteed if $k(\rho + m + 1) + (1-\alpha)\rho > 0$.

Substituting $\frac{\partial Q_j}{\partial t_j}$ into networks' FOC, we have

$$\kappa_M + \kappa_N = p + \frac{p'Q}{m} + \frac{p'Q(1-\alpha)(\rho+m+1)\left[(1-\alpha)\rho + k(m+1)\right]}{m\{k(k-\alpha)(\rho+m+1) - \alpha(1-\alpha)\rho\}}.$$

*Proportional fee*

Under proportional fee, the profit of merchant $i$ is $\pi_i^P = \sum_{j=1}^n \left[\left(1-\tau_j\right) p_j - \kappa_M\right] q_{ij}$. Summing up merchant FOC's over $i$ to obtain network $j$'s average revenue:

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$$\tau_j p_j = p_j + \frac{1}{m}\left(1-\tau_j\right)\frac{\partial p_j}{\partial Q_j}Q_j + \frac{1}{m}\sum_{-j}\left(1-\tau_{-j}\right)\frac{\partial p_{-j}}{\partial Q_j}Q_{-j} - \kappa_M. \tag{24}$$

A system of $n$ such equations will collectively determine $Q_j\left(\tau_j, \tau_{-j}\right)$, $j \in \{1, \ldots, n\}$, where $\tau_{-j}$ is the vector of other networks' proportional fee. Network $j$'s profit is $\Pi_j^P\left(\tau_j\right) = (\tau_j p_j - \kappa_N)Q_j\left(\tau_j, \tau_{-j}\right)$.

Then network FOC's from all $n$ networks will collectively determine a unique equilibrium. Again, we focus on the condition that characterizes the equilibrium virtual output, $Q^P = Q_j^P + \alpha\sum_{-j}Q_{-j}^P = kQ_j^P$ as equilibrium outputs of all networks are identical.

To characterize the equilibrium, we need to substitute $\frac{\partial Q_j}{\partial \tau_j}$ and $\frac{\partial Q_j}{\partial \tau_{-j}} = \frac{\partial Q_{-j}}{\partial \tau_j}$ into networks' FOC's. In what follows, we will show how to obtain $\frac{\partial Q_j}{\partial \tau_j}$ and $\frac{\partial Q_{-j}}{\partial \tau_j}$.

$$\kappa_M = \left(1-\tau_j\right)p_j + \frac{1}{m}\left(1-\tau_j\right)p_j' Q_j + \frac{\alpha}{m}\sum_{-j}\left(1-\tau_{-j}\right)p_{-j}'Q_{-j};$$

$$n\kappa_M = \sum_{j=1}^{n}\left(1-\tau_j\right)p_j + \frac{k}{m}\sum_{j=1}^{n}\left(1-\tau_j\right)p_j'Q_j.$$

By taking derivative with respect to $\tau_j$ on both sides of the equations, we obtain an equation system, from which we can solve $\frac{\partial Q_j}{\partial \tau_j}$ and $\frac{\partial Q}{\partial \tau_j} = \frac{\partial Q_j}{\partial \tau_j} + (k-1)\frac{\partial Q_{-j}}{\partial \tau_j}$. Then, we have

$$\frac{\partial Q_j}{\partial \tau_j} = \frac{p'Q^2}{\kappa_M}\frac{(m\varepsilon-1)\left\{(\rho+m+1)k\left[m\varepsilon(k-\alpha)-(1-\alpha)\right]-\alpha\rho(1-\alpha)(m\varepsilon-1)\right\}}{m(1-\alpha)(\rho+m+1)k\left[(1-\alpha)\rho+k(m+1)\right]};$$

$$\frac{\partial Q_{-j}}{\partial \tau_j} = -\frac{p'Q^2}{\kappa_M}\frac{\alpha(m\varepsilon-1)\{k(\rho+m+1)+(1-\alpha)(m\varepsilon-1)\rho\}}{m(1-\alpha)(\rho+m+1)k\left[(1-\alpha)\rho+k(m+1)\right]};$$

$$\tau = 1 - \frac{m\kappa_M}{mp+p'Q}.$$

To ensure the equilibrium is well-behaved, we need $\frac{\partial Q_j}{\partial \tau_j} < 0$ and $\frac{\partial Q_{-j}}{\partial \tau_j} > 0$, which is guaranteed if $k(\rho+m+1) + (1-\alpha)(m\varepsilon-1)\rho > 0$.

Substituting $\frac{\partial Q_j}{\partial \tau_j}$ and $\frac{\partial Q_j}{\partial \tau_{-j}}$ into networks' FOC, we have

$$\begin{aligned}\kappa_M + \kappa_N = \; & p - \frac{\kappa_M}{m\varepsilon-1} \\ & - \frac{\kappa_M}{m\varepsilon-1}\frac{(1-\alpha)m\left\{(\rho+m+1)\left[\varepsilon(\rho+k)+1\right]-\alpha\rho\left[\varepsilon(\rho+1)+1\right]\right\}}{\left\{(\rho+m+1)k\left[m\varepsilon(k-\alpha)-(1-\alpha)\right]-\alpha(1-\alpha)(m\varepsilon-1)\rho\right\}} \\ & + \frac{p'Q(1-\alpha)\{(\rho+m+1)(km\varepsilon-1)-\alpha\rho(m\varepsilon-1)\}}{\left\{(\rho+m+1)k\left[m\varepsilon(k-\alpha)-(1-\alpha)\right]-\alpha(1-\alpha)(m\varepsilon-1)\rho\right\}}.\end{aligned}$$

A(iv). *Multi-Homing: Welfare*

We can again write the profit function $\Pi_j^P = \Pi_j^U + \Delta\Pi_j$, with $\Pi_j^P$, $\Pi_j^U$ and $\Delta\Pi_j$ being functions of $Q_j$ and $Q_{-j}$, which in turn are functions of $\mathbf{t}$ or $\tau$.

The decomposition is:

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$$
\begin{aligned}
& sign\left\{\frac{\partial \Pi_j^P/\partial \tau_j}{\partial Q_j/\partial \tau_j}-\frac{\partial \Pi_j^U/\partial t_j}{\partial Q_j/\partial t_j}\right\} \\
= \; & sign\left\{\frac{\partial\left(\Pi_j^U+\Delta\Pi_j\right)/\partial \tau_j}{\partial Q_j/\partial \tau_j}-\frac{\partial \Pi_j^U/\partial t_j}{\partial Q_j/\partial t_j}\right\} = sign\left\{\frac{\partial \Delta\Pi_j/\partial \tau_j}{\partial Q_j/\partial \tau_j}+\frac{\partial \Pi_j^U/\partial \tau_j}{\partial Q_j/\partial \tau_j}-\frac{\partial \Pi_j^U/\partial t_j}{\partial Q_j/\partial t_j}\right\} \\
= \; & sign\left\{\frac{\partial \Delta\Pi_j/\partial \tau_j}{\partial Q_j/\partial \tau_j}+\frac{(n-1)\frac{\partial \Pi_j^U}{\partial Q_{-j}}\frac{\partial Q_{-j}}{\partial \tau_j}+\frac{\partial \Pi_j^U}{\partial Q_j}\frac{\partial Q_j}{\partial \tau_j}}{\partial Q_j/\partial \tau_j}-\frac{(n-1)\frac{\partial \Pi_j^U}{\partial Q_{-j}}\frac{\partial Q_{-j}}{\partial t_j}+\frac{\partial \Pi_j^U}{\partial Q_j}\frac{\partial Q_j}{\partial t_j}}{\partial Q_j/\partial t_j}\right\} \\
= \; & sign\left\{\frac{\partial \Delta\Pi_j/\partial \tau_j}{\partial Q_j/\partial \tau_j}+(n-1)\frac{\partial \Pi_j^U}{\partial Q_{-j}}\left(\frac{\partial Q/\partial \tau_j}{\partial Q_j/\partial \tau_j}-\frac{\partial Q/\partial t_j}{\partial Q_j/\partial t_j}\right)\right\} \\
= \; & sign\left\{\frac{\partial\left[-\frac{1}{m}\tau_j p_j' Q_j^2-\frac{\alpha}{m}\sum_{-j}\tau_{-j}p_{-j}'Q_{-j}Q_j\right]/\partial \tau_j}{\partial Q_j/\partial \tau_j}+(n-1)\frac{\partial \Pi_j^U}{\partial Q_{-j}}\left(\frac{\partial Q/\partial \tau_j}{\partial Q_j/\partial \tau_j}-\frac{\partial Q/\partial t_j}{\partial Q_j/\partial t_j}\right)\right\} \\
= \; & sign\left\{\begin{array}{c}
\underbrace{\frac{Q}{\partial Q_j/\partial \tau_j}}_{\text{Concession Effect }(-)} + \underbrace{\tau(\rho+2)\frac{\partial Q/\partial \tau_j}{\partial Q_j/\partial \tau_j}}_{\text{(Direct) Earnings Effect }(+)} + \underbrace{2\tau(k-1)\left(1-\frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j}\right)}_{\text{Competition intensification Effect }(+)} \\
\underbrace{+\,\tau(k-1)\left[(\rho+2)\left(\alpha+(k-\alpha)\frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j}\right)+(k-2\alpha)\left(1-\frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j}\right)\right]}_{\text{(Indirect) Earnings Effect }(+)} \\
+\underbrace{\left\{(k-1)\left[(k+1-\alpha)\rho+(m+1)k\right]\left(\frac{\partial Q_{-j}/\partial t_j}{\partial Q_j/\partial t_j}-\frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j}\right)\right\}}_{\text{Allocation Effect }(+)}
\end{array}\right\}.
\end{aligned}
$$

Note that $\partial Q_j/\partial \tau_j < 0, \partial Q_{-j}/\partial \tau_j > 0, \partial Q/\partial \tau_j < 0$. It is easy to verify that the direct earnings effect and competition intensification effect are positive, and the concession effect is negative. For the indirect earning effect, $\alpha + (k-\alpha)\frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j} = \frac{\partial Q^*_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j}$ where $Q^*_{-j} = Q_{-j} + \alpha\sum_{i\neq -j} Q_i$ is the virtual output in network $-j$. Since the demand is downward sloping, we have $\partial Q^*_{-j}/\partial \tau_j < 0$. Therefore, $\alpha + (k-\alpha)\frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j} > 0$. Together with $1 - \frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j} > 0$, $(k-2\alpha) > 0$, and $(\rho+2) > 0$, this proves that the indirect earning effect is positive.

For the allocation effect, we have $\frac{\partial \Pi_j^U}{\partial Q_{-j}} = -\left[(k+1-\alpha)\rho + (m+1)k\right] < 0$ and $\frac{\partial Q_{-j}/\partial \tau_j}{\partial Q_j/\partial \tau_j} - \frac{\partial Q_{-j}/\partial t_j}{\partial Q_j/\partial t_j} = -\frac{\alpha(1-\alpha)k(\rho+m+1)\left[(1-\alpha)\rho+k(m+1)\right]}{(m\varepsilon-1)\left\{(\rho+m+1)k\left[m\varepsilon(k-\alpha)-(1-\alpha)\right]-\alpha\rho(1-\alpha)(m\varepsilon-1)\right\}\{k(k-\alpha)(\rho+m+1)-\alpha(1-\alpha)\rho\}} < 0.$

By substituting equilibrium condition under unit fee into that under proportional fee, we can verify that $Q^P < Q^U$ if and only if (evaluated at $Q^U$):

$$
\begin{aligned}
& \kappa_N < -\Phi \times \left\{[\varepsilon(\rho+1)+1] + \frac{\varepsilon(k-1)(\rho+m+1)(k-\alpha+1)}{(1-\alpha)[(1-\alpha)\rho+m+1]}\right\} \\
& \equiv \widetilde{\kappa}_N,
\end{aligned}
$$

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where
$$\Phi = \frac{p(Q^U)(1-\alpha)^2(\rho+m+1)[(1-\alpha)\rho+m+1]\left[(1-\alpha)\rho+k(m+1)\right]}{\varepsilon\{(\rho+m+1)k(k-\alpha)-\alpha(1-\alpha)\rho\}\left\{(1-\alpha)[(1-\alpha)\rho+m+1][m\varepsilon(\rho+2)+m-1]+(k-1)\left[km\varepsilon+(1-\alpha)(2m\varepsilon-1)\right](\rho+m+1)\right\}} > 0\,.$$
The condition becomes (19) in the main text given $\kappa_N = 0$.

A(v). *Individual Preferences*

When both networks adopt unit fee, the equilibrium outcomes are:

$$t_i = 3;\ p_i = 4+\kappa_M;\ q_i = 1;\ \Pi_N^{UU} = 3;\ \Pi_M^{UU} = 1.$$

When both networks adopt proportional fee, the equilibrium outcomes are:

$$\begin{aligned}
\tau_i &= 1-\frac{1}{18}\left\{\frac{4\kappa_M\left(9+5\kappa_M\right)}{\sqrt[3]{4}\Delta}-\sqrt[3]{4}\Delta-4\kappa_M\right\};\\
p_i &= \frac{\sqrt[3]{4}\Delta^2-14\kappa_M\Delta-\sqrt[3]{2}\left(18\kappa_M+10\kappa_M^2\right)}{\sqrt[3]{4}\Delta^2+4\kappa_M\Delta-\sqrt[3]{2}\left(18\kappa_M+10\kappa_M^2\right)};\\
q_i &= 1;\\
\Pi_N^{PP}(\kappa_M) &= \frac{(\Gamma+18\Delta)\left[\sqrt[3]{4}\Delta^2-14\kappa_M\Delta-\sqrt[3]{2}\left(18\kappa_M+10\kappa_M^2\right)\right]}{18\Delta\Gamma};\\
\Pi_M^{PP}(\kappa_M) &= \frac{\sqrt[3]{2}\left(18\kappa_M+10\kappa_M^2\right)-4\kappa_M\Delta-\sqrt[3]{4}\Delta^2}{18\Delta},
\end{aligned}$$

where $\Delta = \sqrt[3]{9\sqrt{3\kappa_M^3\left(12+383\kappa_M+104\kappa_M^2+8\kappa_M^3\right)}-38\kappa_M^3-297\kappa_M^2}$, and $\Gamma = \sqrt[3]{4}\Delta^2+4\kappa_M\Delta-\sqrt[3]{2}\left(18\kappa_M+10\kappa_M^2\right)$.

When network 1 adopts proportional fee and network 2 adopts unit fee, the equilibrium outcomes are:

$$\begin{aligned}
\tau_1 &= \frac{27+5\kappa_M+\Omega}{3\left(9+\kappa_M\right)}-\frac{\kappa_M\left(54+29\kappa_M\right)}{3\left(9+\kappa_M\right)\Omega}\\
t_2 &= \frac{\left[\left(9+\kappa_M\right)^2\tau_1^2-\left(9+2\kappa_M\right)\left(9+\kappa_M\right)\tau_1+81+96\kappa_M\right]}{28\kappa_M};\\
p_1 &= 1+\frac{1}{3}\left(t_2+\kappa_M+\frac{2}{1-\tau_1}\kappa_M\right);\\
p_2 &= 1+\frac{1}{3}\left[2\left(t_2+\kappa_M\right)+\frac{1}{1-\tau_1}\kappa_M\right];\\
q_1 &= 1+\frac{1}{3}\left(t_2-\frac{\tau_1}{1-\tau_1}\kappa_M\right);\\
q_2 &= 1+\frac{1}{3}\left(\frac{\tau_1}{1-\tau_1}\kappa_M-t_2\right);\\
\Pi_N^{PU}(\kappa_M) &= \tau_1 p_1 q_1 \text{ (for network 1)};\\
\Pi_N^{UP}(\kappa_M) &= t_2 q_2 \text{ (for network 2)};\\
\Pi_M^{PU}(\kappa_M) &= \left[\left(1-\tau_1\right)p_1-\kappa_M\right]q_1 \text{ (for merchant 1)};\\
\Pi_M^{UP}(\kappa_M) &= \left(p_2-t_2-\kappa_M\right)q_2 \text{ (for merchant 2)};
\end{aligned}$$

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where $\Omega = \sqrt[3]{27\kappa_M\sqrt{\kappa_M\left(216+5109\kappa_M+1618\kappa_M^2+141\kappa_M^3\right)}-\kappa_M^2\left(1863+280\kappa_M\right)}$.

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